PHYSNET

MISN-0-512

FARADAY'S LAW of ELECTROMAGNETIC INDUCTION and MAGNETIC ENERGY DENSITY

Electricity and Magnetism

Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

FARADAY'S LAW of ELECTROMAGNETIC INDUCTION and MAGNETIC ENERGY DENSITY

by

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Title: **Faraday's Law of Electromagnetic Induction and Magnetic Energy Density**

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Input Skills:

1. Vocabulary: ohmic material, steady current, Joule heating (MISN-0-530).
2. Write down Ampere's Law in differential form (MISN-0-609).

Output Skills (Knowledge):

K1. Vocabulary: electromotive force or emf, fundamental equation of electric circuit analysis, magnetic flux, Lenz's law, self-inductance, mutual inductance, magnetostatic energy density.

K2. For a system of magnetically linked, rigid circuits, derive the magnetostatic energy stored in the field structure, given the fundamental equation of electric circuit analysis.

Output Skills (Rule Application):

R1. Use Lenz's law to determine the direction of the current induced in a circuit due to a change in the magnetic field present.

Output Skills (Problem Solving):

S1. Given a system in motion through a stationary magnetic field or a stationary system in the presence of changing magnetic flux, use Faraday's law to determine the induced emf.

S2. Calculate the magnetic energy stored in a system of current-carrying circuits in the presence of linear, isotropic, magnetic materials.

External Resources (Required):

1. J. Reitz, F. Milford and R. Christy, *Foundations of Electromagnetic Theory*, 4th Edition, Addison-Wesley (1993).

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1. Instructor's Notes

1a. Introduction. This Unit introduces the concepts of electromagnetic induction and magnetic energy density. A simple example is the observation that an emf, and hence an electric field, is always produced in a complete circuit as a result of imposing a time-changing magnetic field that is normal to the area enclosed by the circuit.

1b. Flux. To describe this magnetic induction quantitatively and generally, we define the (magnetic) *flux* ϕ through a surface S as:

$$\phi = -\frac{d}{dt}\int \vec{B}\cdot\vec{n}\,dS\,.$$

Here $\vec{n}$ is a unit vector normal to the surface S so $\vec{B}\cdot\vec{n}$ is the component of $\vec{B}$ normal to the surface element dS. Note that flux has the dimensions of magnetic field times area.

1c. Faraday's Law of Induction. Faraday's Law of electromagnetic induction is:

$$\mathcal{E} = -\frac{d\phi}{dt}\,. \tag{1}$$

Eq. (1) can be reexpressed as a partial differential equation involving $\vec{E}$ and $\vec{B}$,

$$\vec{\nabla}\times\vec{E}(\vec{r},t) = -\frac{\partial\vec{B}(\vec{r},t)}{\partial t}\,, \tag{2}$$

where now an explicit time dependence is shown. Eq. (2) is the differential form of Faraday's Law and is one of the famous Maxwell's equations. Eq. (2) shows explicitly that an electric field is associated with a time-varying magnetic induction $\vec{B}$. Because of the explicit dependence on time, the situation is no longer one of electrostatics or magnetostatics but it is a situation better called dynamic electromagnetism.

1d. Not Completely $\vec{E}-\vec{B}$ Symmetric. Since a time-varying magnetic induction is associated with an electric field (actually, the curl of an electric field), it might seem plausible that a time-varying electric field is associated with a non-zero magnetic induction (or, possibly, with the curl of the magnetic induction). Presumably this would happen through Ampere's Law which contains the quantity $\vec{\nabla}\times\vec{B}$. Although no such term as $\partial\vec{E}/\partial t$ appears in Ampere's Law, it must be remembered that the law referred only to a static situation: no time dependence was considered. In fact, it will be shown in MISN-0-513 that when an explicit time dependence is allowed in the vector fields, $\vec{H}$, $\vec{D}$, and $\vec{J}$, and in the scalar field, ρ, in the equations we have seen so far, there is an inconsistency among those equations:

$$\vec{\nabla}\times\vec{H}(t) = \vec{J}(t)\,, \tag{3}$$

$$\vec{\nabla}\cdot\vec{D}(t) = \rho(t)\,, \tag{4}$$

and

$$\frac{\partial\rho(t)}{\partial t} + \vec{\nabla}\cdot\vec{J}(t) = 0\,. \tag{5}$$

Can you find the inconsistency? In Unit MISN-0-513 it will be seen that a modification of Ampere's Law is needed when such an explicit time dependence is allowed.

1e. Magnetostatic Energy Density. Recall from MISN-0-509 that an electrostatic energy density given by

$$\mathcal{W}_{\rm elec} = \frac{1}{2}\vec{D}\cdot\vec{E} \tag{6}$$

could be defined so that the energy W stored in a configuration of charge was given by

$$W = \int \frac{1}{2}\vec{D}\cdot\vec{E}dv\,. \tag{7}$$

The form of Eqs. (6) and (7) suggests that the energy can be viewed as being stored in the fields $\vec{D}$ and $\vec{E}$ produced by the charges in the configuration. Analogously, a magnetostatic energy density is defined by

$$\mathcal{W}_{\rm mag} = \frac{1}{2}\vec{B}\cdot\vec{H}\,, \tag{8}$$

so that the energy W' stored in a configuration of currents is given by:

$$W' = \int \frac{1}{2}\vec{B}\cdot\vec{H}\,dv\,. \tag{9}$$

It becomes very natural then to speak of an electromagnetic energy density, at some space-point and time, given by:

$$\mathcal{W} = \frac{1}{2}(\vec{D} \cdot \vec{E} + \vec{B} \cdot \vec{H}), \tag{10}$$

where the total electromagnetic energy stored in a configuration of charge and currents is given by

$$W = \int \frac{1}{2}(\vec{D} \cdot \vec{E} + \vec{B} \cdot \vec{H}) dv . \tag{11}$$

2. Procedures

1. Read these sections in the text:

   Sec. 11-1 Electromagnetic Induction

   Sec. 11-2 Self-inductance

   Sec. 11-3 Mutual inductance

   Sec. 12-1 Magnetic energy of coupled circuits

   Sec. 12-2 Energy density in the magnetic field

2. Write down or underline the definitions asked for in Output Skill K1.

3. Write down and memorize the derivations asked for in Output Skill K2.

4. Solve these problems:

Problem	**Type**
11 - 2, 11 - 6	Calculation of emf's
11 - 7	Application of Lenz's Law
12 - 1, 12 - 3	Calculation of magnetic energy for simple systems

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