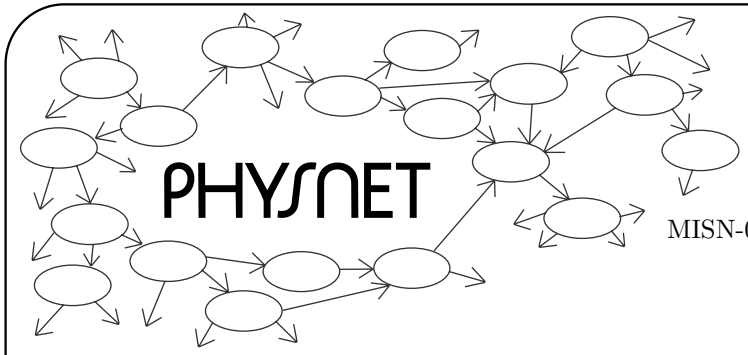

MISN-0-502

RIGID BODIES; THE INERTIA TENSOR

Classical Mechanics

Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

RIGID BODIES; THE INERTIA TENSOR

by

C. P. Frahm

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Title: **Rigid Bodies; the Inertia Tensor**

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Input Skills:

1. Vocabulary: torque, angular momentum, center of mass for a system of particles (MISN-0-494).
2. Write down the transformation properties of an arbitrary tensor in component form (MISN-0-491).

Output Skills (Knowledge):

K1. Show that for a system of particles the torque about a point is equal to the time rate of change of angular momentum about that point provided that at least one of the following is true: (a) the point is the center of mass, (b) the point is at rest or has constant velocity relative to an inertial frame, (c) the position vector of the center of mass relative to the point is parallel to the acceleration of the point.

K2. Define the moment of inertia tensor.

K3. Write down expressions for the rotational kinetic energy and the angular momentum of a rigid body in terms of the inertia tensor.

K4. State and prove the principle axis theorem and the parallel axis theorem for the inertia tensor.

Output Skills (Problem Solving):

S1. Evaluate the inertia tensor for simple rigid bodies, using the principle and parallel axis theorems where necessary.

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1. Introduction

In MISN-0-594 some of the fundamentals of systems of particles were considered. In particular several important theorems dealing with the separation of various physical quantities into a center-of-mass part and a relative motion part were established. That unit lays the groundwork for the material to be covered in this and the subsequent unit – rigid body motion. This unit will be primarily concerned with some kinematical aspects of rigid body motion – in particular the moment of inertia tensor and some of its properties. The next unit will delve into the dynamics of rigid body motion.

2. Procedures

1. Read one of the following:

   Symon, Section 4-2 through p. 160.

   Greenwood, Section on "Arbitrary Reference Point" through to the top of p. 147.

   Be sure to clarify any parts of the analysis that need clarification.

2. Read Sections 11.1 and 11.2 in Marion. Give particular emphasis to the part from eq. (10.13) to the end of Section 11.2.

   ▷ Work problems 11-2 and 11-19 in Marion. Note – for problem 11-19 just obtain the inertia tensor.

3. Reread Section 11.2 of Marion up to Example 11.1. You may find it necessary to read Section 10.2 of Marion to understand Eq. 11.1.

   Read Section 11.3 of Marion. Note that Eq. 11.20a is valid for any choice of the point O (provided that it is fixed in the rigid body), while eq. (11.21) is valid only if point 0 satisfies one of the three conditions given in Output Skill K1.

4. Read section 11.4 of Marion.

   ▷ Exercise - Use the method of Example 11.3 of Marion to find the principle moments of inertia for the rigid body of problem 11-19 in Marion.

   Read Section 11.5 of Marion.

   ▷ Exercise - Find the inertia tensor for the rigid body of problem 11-19 of Marion for a coordinate system oriented parallel to the one given in the problem but with the origin located at the point:

   a. $r = k$, $\theta = 0$;

   b. $r = k$, $\theta = 3\pi/2$.

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