PHYSNET

MISN-0-477

WEAK-FIELD APPROXIMATION AND GRAVITATIONAL WAVES

Relativity

Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

WEAK-FIELD APPROXIMATION AND GRAVITATIONAL WAVES

by

C. P. Frahm

Title: **Weak-Field Approximation and Gravitational Waves**

Author: C. P. Frahm, Dept. of Physics, Illinois State Univ.

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Input Skills:

1. Unknown: assume (MISN-0-476).

Output Skills (Knowledge):

K1. Identify a certain part of the metric tensor with the Newtonian potential by taking the weak-field approximation of the geodesic equations.

K2. Obtain in harmonic coordinates the linearized (weak-field) approximation of Einstein's field equations.

K3. Relate the constant K in Einstein's field equations with Newton's gravitational constant.

K4. Define or explain each of the following: (a) primary motivation which led Einstein to the general theory of relativity, (b) inertial mass and gravitational mass, (c) active and passive gravitational mass, (d) weak, semi-strong and strong equivalence principles, (e) local inertial frame, and the dependence of a frame's extension upon the desired degree of accuracy (f) special relativity as a local theory, (g) bending of light and the gravitational Doppler shift as qualitative consequences of the equivalence principle.

K2. Define or explain: (a) Riemannian space and metric, (b) indefinite metric and signature, (c) geodesics, (d) geodesic separation (define and derive formula for), (e) curvature (for 2-dim and n-dim surfaces), (f) geodesic deviation, (g) isometric spaces.

K3. Outline the basic scheme of general relativity with special emphasis on the roles played by the equivalence principle, by geodesics and by masses.

K4. Derive expressions for (a) the gravitational Doppler shift (and time dilation), (b) the spacetime metric around a spherical mass.

External Resources (Required):

1. W. Rindler, *Essential Relativity*, van Nostrand (1977).

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1. Introduction

Einstein's Field equations, discussed in the previous unit, are highly non-linear and hence difficult to handle in general. However, if the field is weak (in the Newtonian sense) then it is natural to expect that the field equations can be adequately approximated by a set of linear equations which are much easier to work with. This, in fact, is the case as will be demonstrated in this unit. Furthermore, it will be found that this linear approximation has the form of a wave equation and hence predicts the existence of gravitational waves. Finally, if the system is static (rather than dynamic as in the case of wave motion) it will be seen that the linearized (weak-field) equations reduce to Poisson's equation thereby permitting an identification of the constant k in Einstein's field equations.

2. Procedures

1. Read section 4 on pp. 77-79 of Weinberg.

   ▷ Exercise - Fill in any missing details in the analysis leading from the geodesic equations to eq. 3. 4. 5 in Weinberg. Note - Weinberg uses a signature $(-1, 1, 1, 1)$ so that you should obtain

$$g_{oo} = 1 + 2\phi$$

2. Read pp. 251-255 of Weinberg. (For the purposes of this unit it is probably best to omit the material beginning just after eq. 10. 1. 6 and extending through the next paragraph).

   Note - Weinberg's $T_{\mu\nu}$ is the same as Rindler's $T_{\mu\nu}$. Also Weinberg has already identified k with $8\pi G$. You should not make that identification yet. Stick with k for the time being. Your result for eq. 10. 1. 10 should thus read

$$\Box^2 h_{\mu\nu} = -2kS_{\mu\nu}$$

3. ▷ Exercise - Assume a static distribution of dust and show that

$$T^{\mu\nu} = \rho_0 \delta_0^\mu \delta_0^\nu \text{ and } S^{\mu\nu} = \rho_0(\delta_0^\mu \delta_0^\nu - 1/2\eta^{\mu\nu})$$

or equivalently

$$T^{\mu\nu} = \begin{pmatrix} \rho_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \text{ and } S^{\mu\nu} = \frac{\rho_0}{2} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

▷ Exercise - a. Use the preceding result to show that for a static distribution of dust

$$\nabla^2 h_{oo} = k\rho_0$$

b. Then by comparing with Poisson's equation (Rindler's eq. 8.28) and using the result of the exercise in procedure 1 of this unit show that

$$k = 8\pi G$$

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