PHYSNET

MISN-0-467

MINKOWSKI DIAGRAMS, SPACE-TIME INTERVALS

Relativity

Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

MINKOWSKI DIAGRAMS, SPACE-TIME INTERVALS

by

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Title: **Minkowski Diagrams, Space-Time Intervals**

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Input Skills:

1. Vocabulary: relativity principle, spacetime, transformation, event, Lorentz invariant, boost.
2. State the Lorentz transformation for a boost along any coordinate axis (MISN-0-466).

Output Skills (Knowledge):

K1. State the definition of the squared interval between two events. Define the three kinds of intervals. Identify the kind(s) of interval-connecting cause-and-effect-related events. Show that the squared interval is invariant under standard Lorentz transformations.

K2. Draw the Minkowski diagram representing the boost connecting two inertial frames. Establish: (a) the relative orientation of the coordinate axes representing the two frames, (b) the relationship between the scales along the coordinate axes representing the two frames. Indicate: (a) an event's coordinates in the two frames, (b) possible world-lines for particles and for light-signals.

K3. Derive the Lorentz length-contraction. State the length hypothesis. Derive the Einstein time-dilation. State the clock hypothesis.

Output Skills (Problem Solving):

S1. Transform time and space separations of two events from one inertial frame to another. Given the length and orientation of a rod in one frame, find its length and orientation in another.

S2. Given information about two inertial frames, find appropriate information about events as seen from each frame, e.g. event times, event intervals, and the light travel-time between the two origins.

External Resources (Required):

1. W. Rindler, *Essential Relativity*, Van Nostrand (1977).
2. A. P. French, *Special Relativity*, Norton (1968).

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1. Introduction

Some of the most interesting consequences of the postulates of special relativity follow immediately from the Lorentz transformations and concern the relationships between space and time intervals as observed in different inertial frames. Among these are, of course, the famous Lorentz contraction and Einstein time dilation effects. This unit will explore these consequences and will discuss a convenient graphical way of representing many relativistic results via Minkowski diagrams.

2. Procedures

1. Reread the first paragraph of section 2.6 in Rindler.

   Note: $\Delta S^2 \equiv (\Delta S)^2 = (\Delta t)^2 - (\Delta r)^2$ where Δr is the spatial separation of the two events.

   ▷ Show that Δs^2 is invariant under the standard LT given by eq. 2.7 of Rindler.

   Comment: It will be shown in a subsequent unit that Δs^2 is invariant under all Lorentz (or Poincare) transformations. Hence Δs is often called the "invariant interval."

   ▷ Work problems 4-1 and 4-2 in French, p. 120.

   Comment: There are three kinds of intervals depending on whether Δs^2 is positive, zero or negative.

   $\Delta s^2 > 0 \Rightarrow$ time-like interval

   $\Delta s^2 = 0 \Rightarrow$ null or light-like interval

   $\Delta s^2 < 0 \Rightarrow$ space-like interval

   Note: Some authors define the squared interval to be $(\Delta r)^2 - (\Delta t)^2$ in which case the above inequalities are reversed. In either case Δt dominates over Δr in a time-like interval while Δr dominates over Δt in a space-like interval.

   Comment: If two events are related by a space-like interval, then $\Delta t < \Delta r$ or upon inserting c

   $$\Delta t < \frac{\Delta r}{c}$$

   Hence, the time interval between the events is less than the amount of time required for a light signal to propagate between the events. Hence these two events cannot be causally related. On the other hand, if the interval is time-like, there is ample time for a signal to propagate (even at a speed less than c) between the two events. The events may then be (but need not be) cause and effect related.

   (Optional) Read French, pages 117-119.

   Question: Can events related by a null interval be causally related?

   ▷ Work problem 4-18 in French, p. 123.

2. Read Rindler, sections 2.9 and 2.10.

   (Optional) Read French, pages 81-85 (Some of this is also applicable to Output Skill K1).

   ▷ Work problem 3-8 in French, p. 87.

   World-lines are curves in a Minkowski diagram giving the position of a particle vs time (i.e. the life history of the particle). The world-line of a photon (light pulse) propagating along the +x-axis is a straight line in the x-t plane inclined at 45° (since $c = 1$). A free particle with speed v will also have a straight world-line but with greater inclination. Figure 1 shows some possible world-lines for motion along the x-axis.

   Figure 2 shows some impossible world lines for particles.

   Note that since the speed of a particle must always be less than c in an inertial frame the *slope* of its world line must always be greater than 1. The two world lines in the second figure have segments with slopes less than 1.

3. Read Rindler, sections 2.11 and 2.12

   (Optional) Read French, pages 96, 97.

   ▷ Work problem 4-16 in French p. 123.

4. Read Rindler, sections 2.13 and 2.14.

   (Optional) Read French, pages 97-103, 154-159. (omit Method 2 on p. 156)

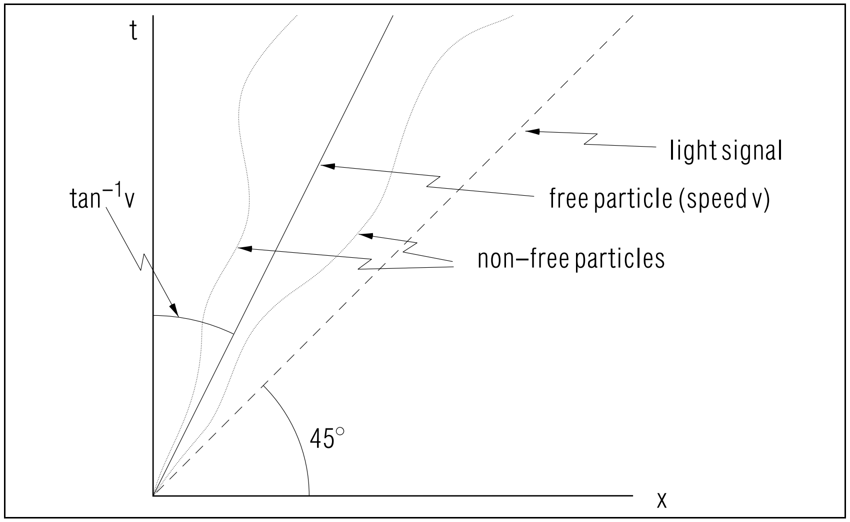

Figure 1. .

Exercise - A free neutron has a lifetime of about 1000 sec. How fast (v) must a neutron travel in order to cross the observable universe ($\sim 10^{10}$ light-years) before decaying? Express your answer by giving the difference $c - v$ to three significant figures.

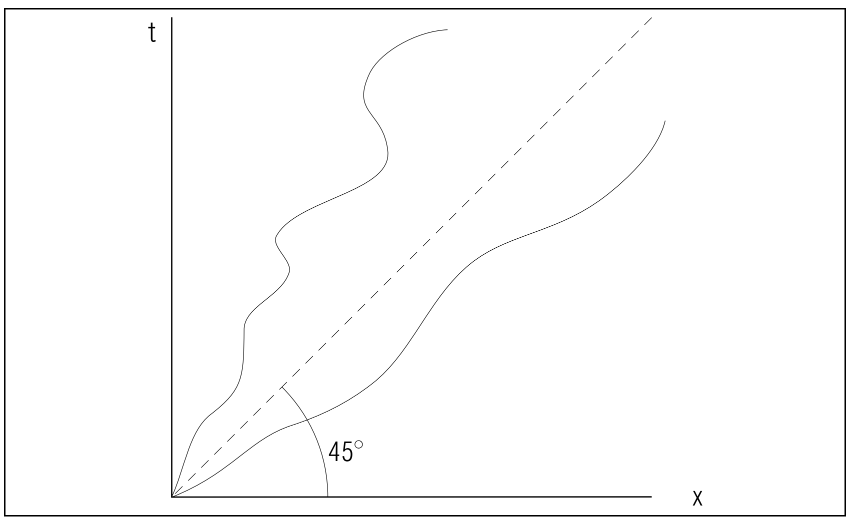

Figure 2. .

5. Work exercises 2-6, 2-7, 2-8, and 2-11 in Rindler, pages 256 - 257

▷ Work exercises 4-6, 4-12 and 4-15 in French, pages 120-122.

Acknowledgments

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