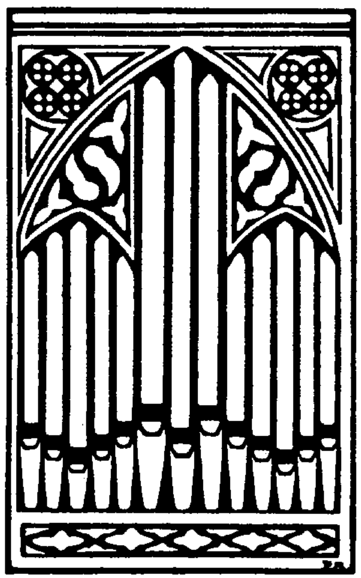
MISN-0-433

STANDING WAVES



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STANDING WAVES

by

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Title: **Standing Waves**

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Input Skills:

1. Vocabulary: amplitude, sinusoidal wave, frequency, wavelength, phase (MISN-0-430).
2. Determine the resultant amplitude of two waves which are one-half cycle out of phase (MISN-0-431).

Output Skills (Knowledge):

K1. Vocabulary: antinode, node, standing wave.

K2. Describe the result of the interference of identical, oppositely moving waves.

K3. Describe the standing waves on a string with fixed boundaries and state the possible values of wavelength and frequency.

K4. State why Fourier analysis is useful.

Output Skills (Problem Solving):

S1. For two waves of the same frequency traveling in opposite directions: (a) relate the positions of the interference maxima or minima to the wavelength or frequency; (b) relate the resultant amplitudes of these maxima or minima to the amplitudes of the individual waves.

S2. For waves traveling between two boundaries, relate the specified conditions at these boundaries to the possible wavelengths or frequencies of the resulting waves.

MISN-0-433

STANDING WAVES

Abstract:
When identical waves move in opposite directions, the resultant wave is called a "standing wave." Such oppositely moving waves again cause characteristic interference maxima and minima in the resultant standing wave. In particular, a standing wave may be produced when identical waves move back and forth between two boundaries (without any supply of outside energy), provided that the wavelength has certain distinct values compatible with the conditions at the boundaries Such standing waves have many applications. For example, the specific wavelengths of standing waves produced in musical instruments determine the frequencies, and thus the pitch, of the tones produced by these instruments. Furthermore, standing waves play a fundamental role in the quantum-theoretical understanding of atomic properties.

SECT.

A INTERFERENCE OF OPPOSITELY MOVING WAVES

► *Standing wave*

Consider two sinusoidal waves w_a and w_b, each having the same wavelength λ and the same amplitude A_1, moving in opposite directions. (For example, the waves might be transverse displacement waves traveling along a long string.) Fig. A-1a shows these waves at several successive times differing by $T/4$, where T is the period of the waves. Fig. A-1b shows the resultant wave equal to the sum of these two waves. Such a wave is called a "standing wave":

Def. **Standing wave**: A wave resulting from the superposition of identical waves moving in opposite directions (A-1)

► *Nodes and antinodes*

The standing wave in Fig. A-1b exhibits pronounced interference effects. For example, at points such as P_1, P_3, P_5, or P_7, the individual waves have always equal magnitudes but opposite signs. Hence the waves interfere there destructively so as to produce a resultant wave of zero amplitude. (Such points, where the resultant wave is always zero, are called "nodes" of the resultant wave.) On the other hand, at points such as P_2, P_4, or P_6, the individual waves have always the same magnitude and the same sign. Hence the waves interfere there constructively to produce a resultant wave of the maximum possible amplitude $2A_1$. (Such points, where the resultant wave has the maximum possible amplitude, are called "antinodes".)

► *Spacing between nodes*

The amplitude of the standing wave exhibits the alternating interference maxima and minima shown in Fig. A-1b because the phase difference between the oppositely moving waves varies from point to point. For example, consider in Fig. A-1 a point P' (such as P_4) at a distance $\lambda/4$ to the right of some other point P (such as P_3). At the point P' the phase of the wave w_a moving to the right is then one-fourth cycle *larger* than at P, while the phase of the wave w_b moving to the left is one-fourth cycle *smaller* than at P. Hence the phase difference between the waves at P' differs from their phase difference at P by one-half cycle.

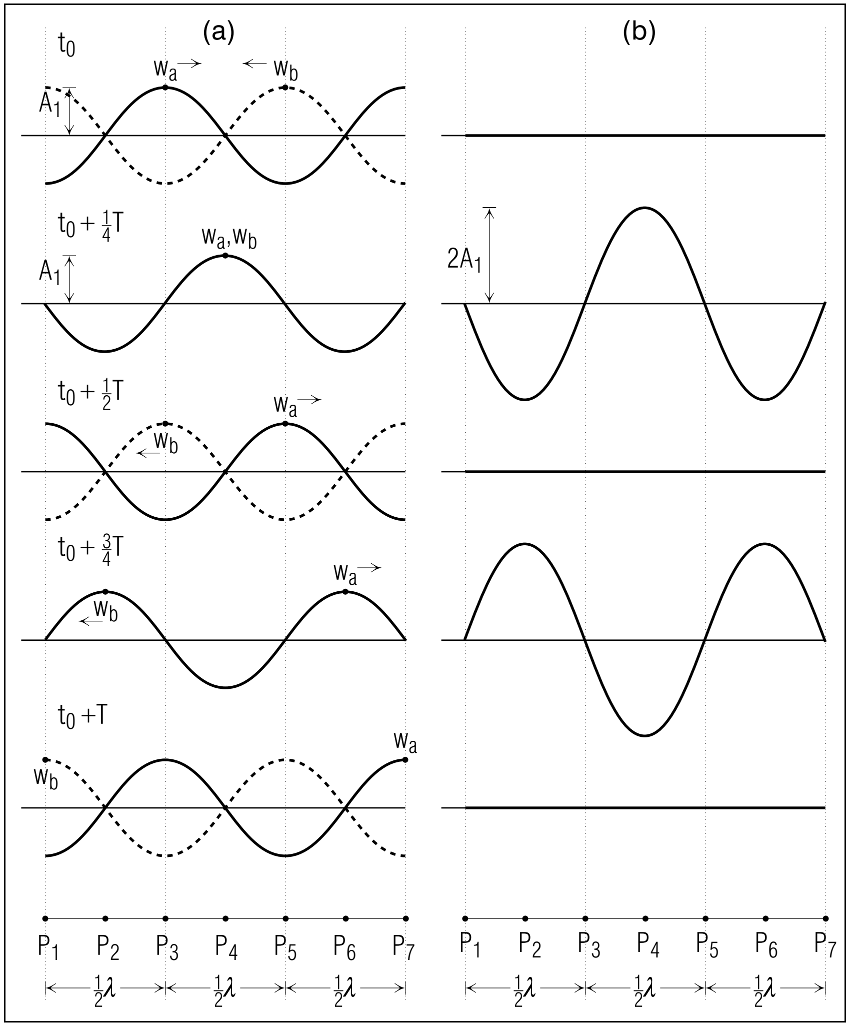

Fig. A-1: (a) Two identical waves, w_a and w_b, moving in opposite directions, shown at five successive times (top to bottom). During a time $T/4$, the wave w_a (solid line) moves a distance $\lambda/4$ to the right while the wave w_b (dashed line) moves a distance $\lambda/4$ to the left. (b) Resultant wave equal to the sum of these waves.

For example, if the waves at P are one-half cycle out of phase so as to interfere there destructively, the waves at P' are in phase so as to interfere there constructively. Thus the separation between a node and an adjacent antinode is $\lambda/4$. (See Fig. A-1b.)

On the other hand, consider in Fig. A-1 a point P' (such as P_5) at a distance $\lambda/2$ to the right of some other point P (such as P_3). At the point P' the phase of the wave w_a moving to the right is then one-half cycle *larger* than at P, while the phase of the wave w_b moving to the left is one-half cycle *smaller* than at P. Hence the phase difference between the waves at P' differs from their phase difference at P by just 1 cycle. Correspondingly, the waves at P' interfere in exactly the same way as they do at P. For example, if the waves interfere destructively at P, they also interfere destructively at P'; and if they interfere constructively at P, they also interfere constructively at P'. Thus we arrive at this conclusion, illustrated in Fig. A-1b:

The separation between adjacent nodes (or antinodes) is $\lambda/2$.

(A-2)

REFLECTION AT A BOUNDARY

► *Node at boundary*

Waves traveling in opposite directions are commonly produced when a wave is reflected from a boundary. Let us consider the simple situation where a wave is incident upon a boundary which is such that the disturbance at the boundary remains always zero. (For example, the wave might be a transverse displacement wave moving along a string toward a "fixed" end of the string, i.e., an end which does not move because it is attached to a wall or some other very massive object. Then the end of the string is a boundary at which the displacement of the string must always be zero.) The incident wave arriving at the boundary then interacts with the boundary so as to produce a "reflected" wave (i.e., a wave moving back in the opposite direction). If the disturbance at the boundary is to remain equal to zero, the wave resulting from the superposition of the incident and reflected waves must then be zero at the boundary, i.e., the resultant wave must have a node at the boundary.

► *Complete reflection*

For example, suppose that the point P_7 in Fig. A-1a is a boundary point where the disturbance remains always equal to zero. When the incident wave w_a arrives at the boundary point P_7, it then produces a reflected wave w_b moving back to the left. Since the resultant wave at the boundary point must be zero, the reflected wave there must cancel the incident wave completely. Hence the reflected wave must have the same amplitude as the incident wave. Since the boundary point is a node of the resultant wave, Def. (A-1) implies that the points at distances $\lambda/2$, $2(\lambda/2)$, $3(\lambda/2)$, ... must also be nodes of the resultant wave. The antinodes are then located halfway between these points. Hence the superposition of the incident and reflected waves results in pronounced interference effects in front of the boundary.

Nodes and Antinodes of Standing Waves (Cap. 1)

$\boxed{\text{A-1}}$ *Oppositely moving sound waves:* Two sound waves, each having a wavelength of 80 cm, move in opposite directions. (a) What is the separation between points where the resultant amplitude of the waves is maximum? What is the separation between points where the resultant amplitude of the waves is minimum? What is the separation between a point where the resultant amplitude is maximum and the next point where the resultant amplitude is minimum? (b) Suppose that each wave has an amplitude described by an excess pressure (above atmospheric pressure) of $4 \times 10^{-4}\,\text{N/m}^2$. What then is the resultant amplitude of the waves at points where this amplitude is maximum and at points where it is minimum? (c) Suppose that one wave has an amplitude corresponding to an excess pressure of $4 \times 10^{-4}\,\text{N/m}^2$ and the other wave has an amplitude corresponding to an excess pressure of $3 \times 10^{-4}\,\text{N/m}^2$. What then is the resultant amplitude at points where this amplitude is maximum and at points where it is minimum? *(Answer: 4)*

$\boxed{\text{A-2}}$ *Standing electromagnetic waves:* An electromagnetic wave, incident perpendicularly upon a metal plate, is completely reflected from this plate. When a small flashlight bulb is moved slowly in front of the plate along a line perpendicular to it, the bulb is observed to become lit and then unlit again, the separation between successive points where the bulb glows most brightly being 5.0 cm. (The bulb glows most intensely when it is located at a point where the amplitude of the electric field is largest.) What is the frequency of the electromagnetic waves incident on the metal plate? *(Answer: 6) (Suggestion: [s-2])*

$\boxed{\text{A-3}}$ *Standing light waves:* A plane light wave, with a wavelength λ is incident perpendicularly upon a plane metallic mirror. The

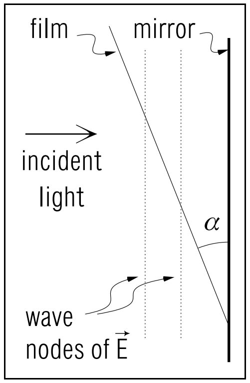

Fig. A-2.

electric field at the mirror must then be nearly zero since the field would otherwise produce enormously large currents in the metal (because it has a very high conductivity). Hence the light reflected from the mirror is such that the total field produced at the mirror by the incident and reflected waves is always nearly zero. A standing light wave can then be detected in front of the mirror by placing there a very thin photograph film making an angle α with the mirror, as indicated in Fig. A-2. The photographic film, after development, should then be blackened most wherever the electric field is largest. (a) What should be the separation between neighboring blackened lines on the film? (b) At what distance along the film, measured from where the film touches the mirror, is the first of these blackened lines? (c) Why is it easier to measure the separation between blackened lines if the angle α is small? (d) What is the numerical value of the observed separation between blackened lines if $\alpha = 2.0°$ and if one uses yellow light with $\lambda = 6.0 \times 10^{-7}$ m? *(Answer: 1) (Suggestion: [s-4])*

SECT.

B STANDING WAVES BETWEEN BOUNDARIES

An especially interesting situation arises when identical waves merely move back and forth repeatedly without energy being supplied from any sources. For example, such a standing wave can be produced when waves move back and forth between two boundaries as a result of repeated reflections from these boundaries.

► *String with fixed ends*

In particular, let us discuss the simple case of transverse displacement waves on a string, of length L, stretched between two fixed boundary points. Such waves can then travel back and forth between these boundary points, being repeatedly reflected at these points. (If the dissipation of the energy of the waves into random internal energy is negligible, the waves keep moving back and forth indefinitely.) Since the boundary points are fixed, the resultant standing wave must then always be zero at these boundary points, i.e., it must have nodes at these points. What then are the wavelengths of the possible standing waves which satisfy the condition that they have nodes at the boundary points?

► *Possible wavelengths*

The possible standing waves can have different numbers of nodes and the separation between adjacent nodes is, by Rule (A-2), always equal to $\lambda/2$. The simplest possible standing wave is then one where the two boundary points are the only nodes, with no other nodes between them. This is the situation illustrated in Fig. B-1a where the wavelength λ is such that $\lambda/2 = L$, the entire length of the string between the boundary points. The next possibility, illustrated in Fig. B-1b, is a standing wave which has one additional node between the boundaries so that the nodes divide the string into 2 parts. Since the distance between nodes is equal to $\lambda/2$, the wavelength in this case is then such that $2(\lambda/2) = L$. The next possibility, illustrated in Fig. B-1c, is a standing wave which has 2 additional nodes between the boundaries so that the nodes divide the string into 3 parts. Since the distance between nodes is equal to $\lambda/2$, the wavelength in this case must then be such that $3(\lambda/2) = L$. And so forth.

To summarize, there can be 0, 1, 2, 3, ... nodes in addition to those at the boundaries of the string. These nodes subdivide the string into n parts, where $n = 1, 2, 3, \ldots$. Since the distance between adjacent nodes is equal to $\lambda/2$, the wavelength λ of the corresponding standing waves

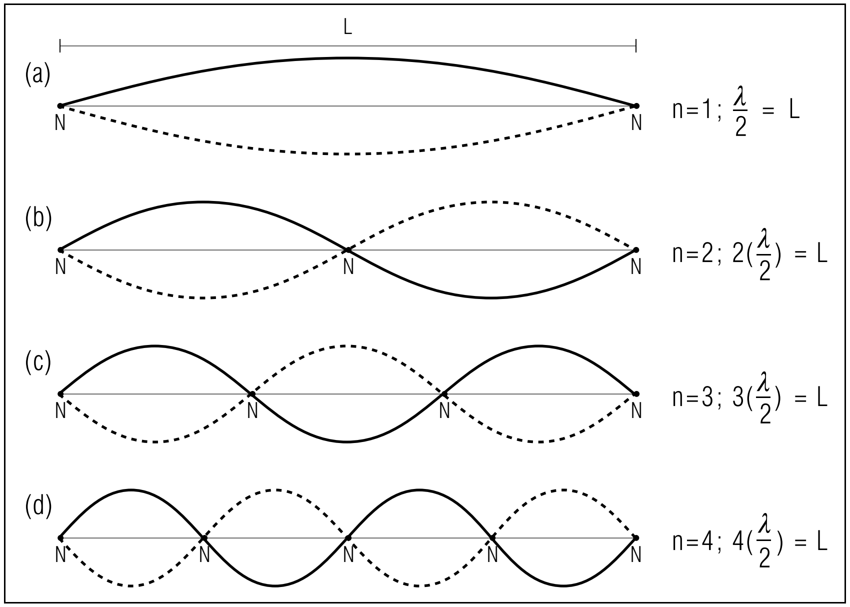

Fig. B-1: Possible standing waves on a string fixed at both ends. Each diagram indicates the standing wave at one time (solid line) and half a period later (dashed line). The nodes are indicated by the letter N.

must then be such that

$$\boxed{n\left(\frac{\lambda}{2}\right) = L, \ \text{where } n = 1, 2, 3, \ldots} \tag{B-1}$$

or

$$\lambda = \frac{2L}{n} \tag{B-2}$$

► *Possible frequencies*

The frequency ν of each wave is related to its wavelength λ so that $\lambda\nu = V$, the speed of the waves along the string. Hence the various possible wavelengths specified by Eq. (B-2) are related to the corresponding frequencies of these waves so that

$$\boxed{\nu = \frac{V}{\lambda} = n\left(\frac{V}{2L}\right)} \tag{B-3}$$

► *Summary*

The condition that the boundary points of the string remain fixed implies that there can only be certain possible standing waves (or "modes of vibration" of the string) resulting from waves traveling back and forth along the string. Each of these waves is characterized by a distinct wavelength and corresponding frequency. The first possible standing wave, labeled by $n = 1$ in Fig. B-1, is one where the string vibrates as a whole with no nodes between its boundaries. This standing wave has the largest possible wavelength λ_1, and correspondingly smallest possible frequency ν_1, given by Eq. (B-2) and Eq. (B-3) with $n = 1$. Thus

$$\lambda_1 = 2L,\ \nu_1 = \frac{V}{2L} \qquad \text{(B-4)}$$

The other standing waves, labeled by $n = 2, 3, 4, \ldots$, have successively more nodes and thus correspondingly smaller wavelengths and larger frequencies given by Eq. (B-2) and Eq. (B-3). Thus

$$\lambda = \frac{\lambda_1}{n},\ \nu = n\nu_1 \qquad \text{(B-5)}$$

The frequencies of all the possible standing waves are thus simply integral multiples of the smallest possible frequency ν_1 characterizing the first standing wave.

APPLICATION TO MUSICAL INSTRUMENTS

► *String instruments*

The preceding discussion of standing waves has immediate applications to string instruments (such as violins or guitars). In such an instrument the vibrations of a string stretched between two fixed points are used to produce various tones. The length of the string determines the wavelength, and thus the corresponding frequency, of any mode of vibration (i.e., standing wave) set up in the string. This frequency of vibration of the string is then equal to the frequency of the sound waves produced by the string in the surrounding air. In general, the string may vibrate in a complex fashion so that its vibrations may be described as the sum of several modes of vibration occurring simultaneously. (See Sec. C.) The frequency of the standing wave of lowest frequency (the "fundamental" mode of vibration) determines then the perceived *pitch* of the correspond-

ing sound wave in air, while the frequencies of the standing waves of higher frequency (the "harmonic" modes of vibration) determine the perceived tonal *quality* of the sound.

► *Changing pitch*

In order to change the pitch of a tone, a musician places a finger on the string so as to shorten the length L of the string which is left free to vibrate. By Eq. (B-4), the wavelength of the fundamental mode of vibration of the string is thus decreased and the frequency of vibration of the string is correspondingly increased. Thus the pitch of the produced sound is also increased. For example, suppose that the musician wants to produce a tone which has (according to musical terminology) a pitch an "octave" higher than another tone. Such a tone is caused by a sound wave having a frequency twice as large as that of the other tone. To produce this tone, the musician places his finger in the precise middle of the string, thus shortening by half the length of the string free to vibrate. Correspondingly the frequency of vibration of this shortened string is then twice as large as that of the original entire string.

► *Wind instruments*

A wind instrument (such as a flute, trombone, or organ) produces tones by setting up standing sound waves in a column of air within a tube. (Such standing waves can be analyzed in a manner similar to standing waves on a string.) The frequencies of vibrations of these standing waves determine the pitch of the sound produced by the wind instrument. Thus a shortened column of vibrating air produces a standing wave with a smaller wavelength and correspondingly larger frequency, and thus produces sound of a higher pitch. (For example, a short organ pipe produces a tone of a higher pitch than a long organ pipe.)

STANDING WAVES IN SEVERAL DIMENSIONS

Standing waves produced on a string are particularly simple since all the waves travel parallel to a single direction. Waves traveling along surfaces or in three-dimensional space can similarly produce more complex standing waves in several dimensions. For example, consider a drum consisting of a circular membrane stretched so that it is fastened around its circumference. Then elastic displacement waves can travel back and forth over the surface of this membrane. The resulting standing waves on the membrane must then be such as to satisfy the condition that the disturbance at the fixed circular boundary of the membrane is zero, i.e., that this boundary be a node. This condition then allows again only

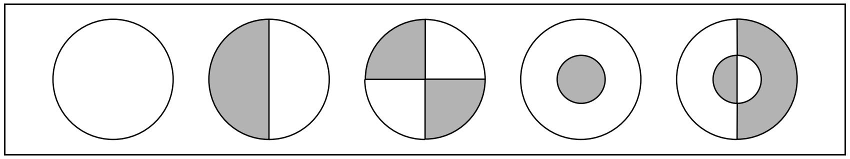

Fig. B-2: Some of the possible standing waves of a vibrating circular membrane. The lines indicate the nodes. At any instant, the displacement of a shaded region is opposite to that of an unshaded region.

certain possible standing waves corresponding to different numbers and kinds of nodal lines along which the resultant wave remains zero. Fig. B-2 illustrates a few of these possible standing waves (or modes of vibration) of such a membrane.

Standing Waves between Boundaries (Cap. 2)

B-1 *Standing electromagnetic waves between parallel metal plates:* Standing eletromagnetic waves are set up between two parallel metal plates separated by a distance of 30 cm. The resultant electric field must then be nearly zero at each of these plates (since the current produced in each of these highly conducting plates would otherwise be gigantically large). (a) What then are the wavelengths of the four standing waves with the longest possible wavelengths? (b) What are the corresponding frequencies of these standing waves (assuming that the space between the plates is a vacuum)? *(Answer: 5)*

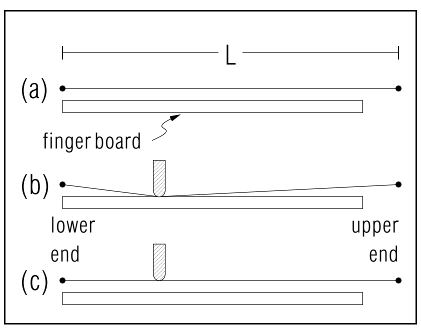

Fig. B-3.

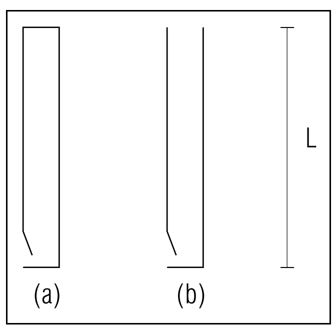

Fig. B-4.

B-2 *Producing tones on a violin:* The "A-string" of a violin, when properly stretched between its two fixed ends separated by a distance L, vibrates so that its "fundamental frequency" (i.e., the frequency of the standing wave of lowest possible frequency) is $\nu = 440\,\text{Hz}$ and thus produces the tone with the pitch of "standard A." (See Fig. B-3a.) To produce a tone of higher pitch, the violinist places a finger at an appropriate place on the string. If the finger there is pressed against the "fingerboard" (as shown in Fig. B-3b), only the upper part of the string (between the finger and the upper end of the string) then vibrates when the string is bowed near its upper end. (a) The tone of A that has, in musical language, a pitch an "octave" above the original A, has a fundamental frequency 2 times as large as ν. At what distance from the lower end of the string must the violinist place his finger so as to produce this tone? Express your answer in terms of L. (b) The tone of "E," which has a pitch a "fifth" above that of the original A, has a frequency $(3/2)\nu$. At what distance from the *lower* end of the string must the violinist place his finger to produce this tone? *(Answer: 3) (Suggestion: [s-1])*

B-3 *Producing "harmonic" tones:* A violinist may also touch a string only lightly with his finger (as shown in Fig. B-3c), without pressing the string to the fingerboard. Then the string still vibrates as a whole, but must have a node at the position of the finger. (Since modes of vibration without a node at this position are suppressed, the tone thus produced has an ethereal quality and is called a "harmonic" tone by violinists.) The perceived pitch of this tone corresponds then to the fundamental frequency (i.e., the lowest possible frequency) of the standing wave with a node at the position of the finger. (a) Suppose that the violinist touches his finger only lightly at the same position as that described in part a of problem B-2. What then is the fundamental frequency of the resulting tone? Express your answer in terms of the original frequency ν of the undisturbed string and also in terms of the frequency $\nu_a = 2\nu$ produced when the finger is pressed to the fingerboard. (b) Suppose that the violinist touches his finger only lightly at the same position as that described in part (b) of problem B-2. What then is the fundamental frequency of the resulting tone? Express your answer in terms of the original frequency ν of the undisturbed string and also in terms of the frequency $\nu_b = (3/2)\nu$ produced when the finger is pressed to the fingerboard. *(Answer: 8) (Suggestion: [s-5])*

B-4 *Closed and open organ pipes:* Tones are produced in an organ by blowing into the bottom of pipes and thus setting up standing sound waves of the air in the pipes. (a) One type of organ pipe is closed at both ends, as shown in Fig. B-4a The net displacement of the air must then be zero (i.e., the resultant standing sound wave must have nodes) at each end of the pipe. If the length of the organ pipe is L, what are the wavelengths of the standing sound waves with the longest three possible wavelengths? If the speed of sound in air is V, what are the corresponding frequencies of the sound produced by these three standing waves? (b) Another type of organ pipe is closed at one end and open at its other end, as shown in Fig. B-4b. The net displacement of the vibrating air must then be zero at the closed end of the pipe and is approximately maximum at the open end of the pipe (i.e., the resulting standing wave has a node at the closed end and an antinode at its open end). If the length of the pipe is L, what are then the wavelengths of the standing sound waves with the longest three possible wavelengths? What are the corresponding frequencies of the sound produced by these three standing waves? *(Answer: 2) (Suggestion: [s-3])*

B-5 *Pitch produced by an organ pipe:* The pitch of the sound produced by an organ pipe corresponds to the fundamental frequency (i.e., the lowest possible frequency) of the standing sound wave produced in the pipe. (a) The speed of sound in air is 340 m/s. What then must be the length of an organ pipe, closed at both ends, which produces the pitch of A corresponding to the frequency of 440 Hz? (b) What must be the length of an organ pipe, closed at one end and open at the other, which produces this same pitch? (c) To produce tones of higher pitch, should organ pipes be longer or shorter? (d) The speed of sound in air increases slightly with increasing temperature. As a result, does the pitch of the tone produced by an organ pipe of given length increase or decrease? *(Answer: 10)*

More practice for this Capability: [p-1], [p-2], [p-3]

SECT.

C ANALYSIS OF COMPLEX WAVES

Our discussion throughout the preceding units has dealt almost entirely with sinusoidal waves. How can this discussion be extended to deal with more complex waves which are not simply sinusoidal?

► *Fourier analysis*

The answer to this question is provided by the following mathematical result, first proved by the French mathematician and physicist J.B.J. Fourier (1768-1830):

Any disturbance can always be expressed as a sum of sinusoidal disturbances of all possible frequencies, with appropriately chosen amplitudes and phases.	(C-1)

This result is extremely useful because it permits one to express any wave, no matter how complex, as a sum of simple sinusoidal waves of various frequencies. By this procedure, called "Fourier analysis," it is possible to discuss any complex wave phenomenon by using merely a knowledge of the behavior of simple sinusoidal waves.*

* The result that any disturbance can be expressed as a sum of component sinusoidal disturbances of various frequencies is analogous to the familiar result that any vector can be expressed as a sum of component vectors along various directions.

► *Plausibility argument*

Let us indicate why the remarkable result in Rule (C-1) is plausible (although a mathematical proof is beyond the scope of this book). Consider at some fixed point any disturbance (or wave) w which varies in the course of time in any manner, as indicated in Fig. C-1a. We shall now try to express this disturbance as a sum of simpler disturbances:

(1) The disturbance w in Fig. C-1a can be expressed as a sum of successive "pulses" (i.e., disturbances of very short duration), where each pulse at any time has a value equal to that of the original disturbance w at this time. (See Fig. C-1b.)

(2) Consider then any such pulse occurring at some time t_0, as indicated in Fig. C-1c. Suppose that we take very many sinusoidal distur-

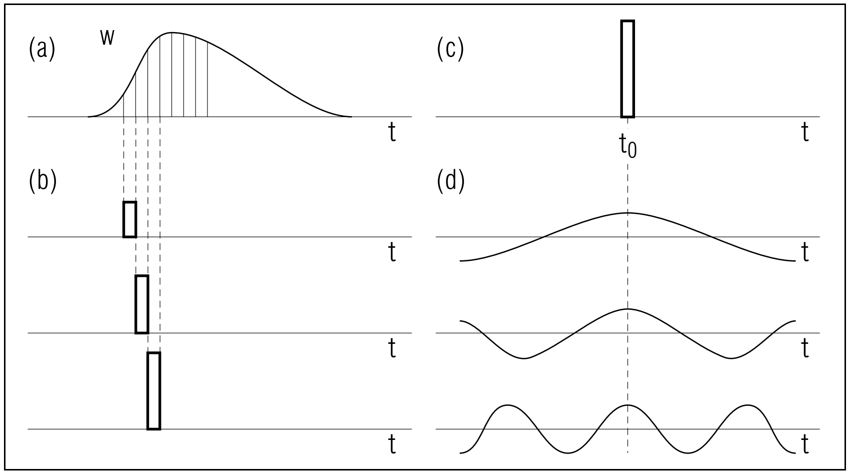

Fig. C-1: A disturbance expressed as a sum of sinusoidal disturbances. (a) A disturbance at a fixed point varying with the time t. (b) This disturbance expressed as a sum of many pulses such as those indicated. (c) A single one of these pulses. (d) This pulse expressed as a sum of many sinusoidal disturbances such as those indicated.

bances, of all possible frequencies, with phases chosen so that each of these disturbances has its maximum value at the time t_0. (See Fig. C-1d.) Then all these sinusoidal disturbances have the same sign at the time t_0 so that their sum there is large. (In other words, the disturbances interfere constructively at this time.) But since the disturbances have different frequencies, some of these disturbances are positive and others are negative at any time different from t_0. Hence the sum of many such disturbances is, at any time different from t_0, negligibly small compared to their sum at t_0. (In other words, the disturbances interfere destructively at all times other than t_0.) Thus the sum of all these sinusoidal disturbances approximates a pulse which is appreciable only at the time t_0 and negligible at all other times. If the amplitudes of the sinusoidal disturbances in Fig. C-1d are appropriately chosen, their sum can then be made to approximate the pulse in Fig. C-1c.

According to the preceding comments, any disturbance can be expressed as a sum of successive pulses; in turn, any such pulse can then be expressed as a sum of sinusoidal waves of all possible frequencies. The combination of these arguments leads thus to the result, in Rule (C-1),

that any disturbance can be expressed as a sum of sinusoidal waves of all possible frequencies.

APPLICATIONS

► *Musical tones*

The sound wave produced by a vibrating tuning fork consists predominantly of a wave of a single frequency. But the sounds produced by the human voice or by musical instruments (such as violins, guitars, pianos, bells,...) are caused by complex vibrations of strings or other objects. Hence these sounds are due to complex sound waves which can, according to Rule (C-1), be analyzed into a sum of sinusoidal waves of many different frequencies. For example, the tone corresponding to the note of "A" produced by a violin A-string seems different from the same tone of "A" produced by an oboe. When the sound waves corresponding to each of these notes are analyzed into sums of component sinusoidal waves, the component wave of lowest frequency has in each case the same frequency of 440 hertz. This is the reason why the tones from both instruments are perceived to have the same pitch. But since the amplitudes of the component sound waves of higher frequencies are different for the two instruments, the quality of the tones produced by these instruments is perceived to be different.

► *Audio amplifiers*

An audio amplifier used for the electronic recording or playing of music is often specified by its "frequency response." For example, an amplifier is said to have a frequency response from 25 Hz to 18,000 Hz if it amplifies equally all sinusoidal disturbances having frequencies between these values. If such an amplifier is used to amplify any tone, no matter how complex, it then amplifies equally its component sound waves for all frequencies audible to the human ear. Hence such an amplifier is called a "high-fidelity" amplifier. By contrast, an amplifier used in a dictating machine might have a much more limited frequency response from 100 Hz to 6000 Hz. Such an amplifier then amplifies equally sounds with frequencies in this range, but not outside this range. Since some of the sinusoidal waves of frequencies audible to the human ear are not amplified by the amplifier, the tone quality of sound emerging from the amplifier is distorted compared to the tone quality of the original unamplified sound. Such a distortion may be quite acceptable for an adequate understanding of human speech, but would be objectionable for the playing of good music.

► *Radios*

Since many radio stations broadcast electromagnetic waves simultaneously, the total electric field produced by all these waves at any point varies with time in a complex way. But a radio placed at such a point can be tuned so as to amplify only a specified sinusoidal component of this electric field. Thus the radio can be used to select only that component of the electric field having the frequency of the electromagnetic wave sent out by a particular station.

► *Complex standing waves*

The result, Rule (C-1), applies equally well to standing waves. For example, after a guitar string has been plucked, the string vibrates in quite a complex way. But this complex vibration can be expressed as a sum of all the possible sinusoidal standing waves, of different frequencies, which can be set up on such a string (i.e., as a sum of standing waves, such as those illustrated in Fig. B-1, each with an appropriately chosen amplitude and phase).

SECT. D SUMMARY

DEFINITIONS

Standing wave; Def. (A-1)

IMPORTANT RESULTS

Interference of identical oppositely moving waves: Rule (A-2)

Nodes (or antinodes) are separated by $\lambda/2$

Standing waves on a string with fixed boundaries: Eq. (B-1), Eq. (B-2), Eq. (B-3)

λ such that $n(\lambda/2) = L$, where $n = 1, 2, 3, \ldots$ Hence $\lambda = (2L)/n$ and $\nu = V/\lambda = n(V/2L)$

Fourier analysis: Rule (C-1)

Any disturbance can be expressed as a sum of sinusoidal disturbances of all possible frequencies.

NEW CAPABILITIES

(1) For two waves of the same frequency traveling in opposite directions, (a) relate the positions of the interference maxima and minima to the wavelength or frequency; (b) relate the resultant amplitudes of these maxima or minima to the amplitudes of the individual waves. (Sec. A)

(2) For waves traveling between two boundaries, relate the specified conditions at these boundaries to the possible wavelengths or frequencies of the resulting standing waves. (Sec. B; [p-1], [p-2], [p-3])

D-1 *Standing-wave measurement of sound speed in a gas (Cap. 2):* The speed of sound in a gas can be measured by using a horizontal tube containing the gas and a small amount of powder spread over the bottom surface of the tube. One of the closed ends of this tube consists of the diaphragm of a loudspeaker producing sound waves of a fixed frequency ν. The other end of the tube consists of a movable piston which can be adjusted in position until a standing sound wave is produced in the tube. The existence of such a standing wave can be detected by observing that the powder in the tube then piles up in small ridges at those places where there is a node of the displacement of the gas so that the powder there remains undisturbed. (See Fig. D-1.) The distance d between two such neighboring ridges (i.e., between two nodes of the standing wave)

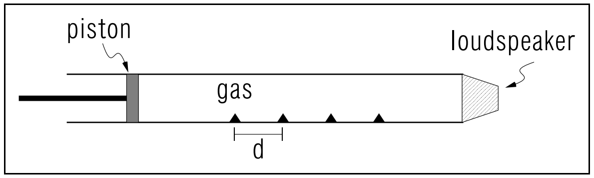

Fig. D-1.

can then be measured with a ruler. Express the speed V of sound in the gas inside the tube in terms of the frequency ν of the loudspeaker and the measured distance d. *(Answer: 7)*

SECT.

E PROBLEMS

E-1 *Standing waves produced by varying the tension in a string:* Transverse standing waves in a string can be easily demonstrated by attaching one end of the string to a tuning fork vibrating at some fixed frequency ν and by attaching the other end of the string to a peg (or similar device) which allows one to vary the tension in the string. Suppose that the string vibrates so that the observed standing wave has only one loop (i.e., no nodes except at the ends of the string). If one wants to demonstrate standing waves with more loops (i.e., with more nodes), should the tension of the string be increased or decreased? *(Answer: 12) (Suggestion: [s-8])*

E-2 *Relation between pitch and properties of a string:* In text problem (H-1) of Unit 430 you showed, by arguments of unit consistency, that the speed V of transverse waves along a string is proportional to $(F_t/m')^{1/2}$ where F_t is the magnitude of the tension force acting on the string and m' is the mass per unit length of the string. (a) If the magnitude of the tension force on a string is increased, does the pitch of the tone produced by the string increase or decrease? b) In string instruments (such as the violin or the piano) certain strings are covered by a winding of metal wire. Is this done to increase or decrease the pitch of the tone produced by such a string when it is subjected to a given tension force? (c) If the original magnitude of the tension force on a string is F_t, what must be the magnitude of the tension force required so that frequency of the sound produced by the string is 3/2 larger than the original frequency (i.e., so that the new pitch is one "fifth" higher than the original pitch)? *(Answer: 9)*

E-3 *Resonance and speed of sound:* A particular standing wave is usually set up in a system whenever the system is excited by an outside source at a frequency equal to that of this particular standing wave. (This phenomenon is called "resonance".) For example, Figure E-1 illustrates a tuning fork vibrating at a frequency of 1056 Hz, placed above an open vertical tube filled with water. If water is gradually removed so as to lower the water level in the tube, resonance can then be detected at various heights of the water level because the corresponding standing sound waves (set up in the column of air between the water surface and the open top end of the tube) lead to a sound louder than that produced by the tuning fork alone. Suppose that the first such resonance occurs

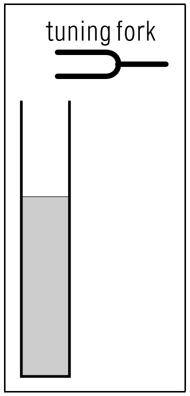

Fig. E-1.

when the water level is 7.90 cm below the open end of the tube. What then is the speed of sound in the air of the tube? *(Answer: 14) (Suggestion: [s-6])*

E-4 *Resonances in an air column:* Suppose that the speed of sound in air is 330 m/s and that the tuning fork placed above the water-filled tube of Fig. E-1 vibrates at a frequency of 660 Hz (thus producing the tone of E). At what possible distances of the water level below the open top end of the tube does one then detect resonance if the length of the tube is 1 meter? *(Answer: 16)*

E-5 *Ordinary and "harmonic" tones produced by a string:* An unfingered violin string, of length L_0, produces a tone corresponding to a sound frequency ν_0. (a) What then is the frequency ν of the ordinary sound produced when the string is pressed against the fingerboard at distance L_0/n from its lower end (where n is some integer such as 1,2,3,...)? (b) What is the frequency ν' of the "harmonic" sound produced when the finger is touched lightly at the same point of the string? Express your answer in terms of the frequency ν_0 of the unfingered string and also in terms of the frequency ν of the ordinary sound introduced by the finger placed at this position. *(Answer: 13) (Suggestion: [s-9])*

PRACTICE PROBLEMS

p-1 *STANDING WAVES BETWEEN BOUNDARIES (CAP. 2): Tone production on a violin string* A violin D string, having a length of 33 cm, sounds the tone of D (corresponding to a frequency of 293.3 Hz) when it is played without fingering. At what distance from the lower end of the string must one place a finger so as to produce the tone of F having a frequency of 352 Hz? *(Answer: 53) (Suggestion: Review text problem B-2 and see Fig. B-3 for the definition of "lower end of string.")*

p-2 *STANDING WAVES BETWEEN BOUNDARIES (CAP. 2): Speed of waves on a guitar string* The B string of a guitar has a length of 60 cm and produces the tone of B having a frequency of 247 Hz. What then is the speed of transverse waves traveling along this string? *(Answer: 57) (Suggestion: See [s-7].)*

p-3 *STANDING WAVES BETWEEN BOUNDARIES (CAP. 2): String instruments and musical intervals* A string of a string instrument (such as a violin or guitar) has a length L and produces a tone corresponding to a frequency ν. In musical language, the tone with a pitch a "fifth" higher has then a frequency $(3/2)\nu$; the tone with a pitch a "fourth" higher has a frequency $(4/3)\nu$; and the tone with a pitch a "major third" higher has a frequency $(5/4)\nu$. At what distance from the lower end of the string must one place a finger to produce each of these three tones? Express your answer in terms of L. *(Answer: 56) (Suggestion: Review text problem B-2.)*

SUGGESTIONS

s-1 *(Text problem B-2):* Suppose that the string, of length L, vibrates with a fundamental frequency corresponding to the standing wave of lowest possible frequency (or longest possible wavelength). Suppose that the string is now shortened so that its new length is L', while the tension of the string (and thus the speed of waves along the string) remains unchanged. What then is the fundamental frequency ν' of this shortened string? *(Answer: 54)*

s-2 *(Text problem A-2):* In front of the plate there is a standing wave resulting from the incident wave and the reflected wave traveling back in the opposite direction. What is the separation between successive antinodes of the electric field in this standing wave? What then is the wavelength of the electromagnetic waves? What then is the frequency of these waves, since the speed of electromagnetic waves in vacuum (or air) is 3×10^8 m/s?

s-3 *(Text problem B-4):* Sound waves travel back and forth in the air inside each pipe. Sketch the resulting possible standing displacement waves which can exist at any instant so as to satisfy the specified conditions at the ends of the pipe. Remember that the spacing between successive nodes is $\lambda/2$, and that the spacing between a node and a neighboring antinode is $\lambda/4$. (a) If the pipe is closed at both ends, what is the wavelength of the standing wave which has no nodes except at both ends of the pipe? (b) If the pipe is closed at one end and open at the other end, what is the wavelength of the standing wave which has no nodes, except a node at the closed end and an antinode at the open end? *(Answer: 55)*

s-4 *(Text problem A-3):* The resultant electric field has a node at the plane mirror. (a) What then is the distance between the mirror and the next parallel plane where the resultant field has an antinode (i.e., where the amplitude of this field is largest and thus produces maximum blackening of the film)? (b) What then is the distance between this plane, corresponding to an antinode,and the next plane corresponding to an antinode? (c) The film is blackened wherever it intersects a plane corresponding to an antinode of the resultant electric field. If two such neighboring planes are separated by distance L, and the film makes an angle α with any such plane, what is the distance along the film between the lines where the film intersects these planes? Draw a diagram and use simple trigonometry. *(Answer: 52)*

s-5 *(Text problem B-3):* Sketch the possible modes of vibration of the entire string,of length L,and pick out the first possible mode where there is a node at the position of the finger. What then is the wavelength of this standing wave, expressed in terms of L? What then is the corresponding frequency of vibration of the string?

s-6 *(Text problem E-3):* Review the discussion of open organ pipes in problem B-4.

s-7 *(Practice problem [p-2]):* If the frequency of the sound waves produced by the string is 247 Hz, what is the frequency of vibration of the standing transverse wave of lowest frequency produced on this string? What is the wavelength of this standing wave if the length of the string is 60 cm? What then is the speed of the waves traveling along the string?

s-8 *(Text problem E-1):* (a) If one wants to observe more nodes, should the wavelength of waves traveling along the string be larger or smaller? (b) Since the frequency of vibration remains unchanged, should the speed of transverse waves along the string then be made larger or smaller? (c) Is this achieved by making the tension in the string larger or smaller? *(Answer: 51)*

s-9 *(Text problem E-5):* Review text problems B-2 and B-3.

ANSWERS TO PROBLEMS

1. a. $\frac{\lambda}{2}/\sin\alpha$
   b. $\frac{\lambda}{4}/\sin\alpha$
   c. The separation is then larger.
   d. 8.6×10^{-6} m
2. a. $2L$, L, $(2/3)L$; $(V/2L)$, $2(V/2L)$, $3(V/2L)$
   b. $4L$, $(4/3)L$, $(4/5)L$; $(V/4L)$, $3(V/4L)$, $5(V/4L)$
3. a. $L/2$
   b. $L/3$ from the lower end (answer is NOT $2L/3$)
4. a. 40 cm, 40 cm, 20 cm
   b. $8 \times 10^{-4}\,\mathrm{N/m^2}$, 0
   c. $7 \times 10^{-4}\,\mathrm{N/m^2}$, $1 \times 10^{-4}\,\mathrm{N/m^2}$
5. a. 60 cm, 30 cm, 20 cm, 15 cm
   b. 5.0×10^{8} Hz, 1.0×10^{9} Hz, 1.5×10^{9} Hz, 2.0×10^{9} Hz
6. 3×10^{9} Hz
7. $V = 2\nu d$
8. a. $2\nu = \nu_a$
   b. $3\nu = 2\nu_b$
9. a. increase
   b. decrease
   c. 2.25 F_t
10. a. 0.39 m
    b. 0.19 m
    c. shorter
    d. increase
11. (not used)
12. decreased
13. a. $\nu = \dfrac{n}{n-1}\nu_0$

b. $\nu' = n\nu_0 = (n-1)\nu$

14. 334 m/s
15. (not used)
16. 12.5 cm, 37.5 cm, 62.5 cm, 87.5cm
17. (not used)
51. a. smaller
    b. smaller
    c. smaller
52. a. $\lambda/4$
    b. $\lambda/2$
    c. $L/\sin\alpha$
53. 5.5 cm
54. $\nu' = \nu(L/L')$
55. a. 2L
    b. 4L
56. $L/3$, $L/4$, $L/5$
57. 296 m/s
58. (not used)

MODEL EXAM

GIVEN INFORMATION: speed of sound in air = 340 meter/second

1. **Speed of sound in methane gas.** A tube into which methane gas is slowly flowing has a number of closely-spaced holes along its upper surface. The gas flowing from these holes is lighted. When sound waves pass in opposite directions along the interior of the tube, a standing wave is set up. At the pressure antinodes of the standing wave, the flames above the tube are larger than at other points.
   a. If the traveling waves have a frequency of 1350 hertz, the large flames are separated by 0.16 meter. What is the speed of sound in methane?
   b. What is a pressure antinode?
2. **Amplitude in a standing wave.** A radio wave is perpendicularly incident on a wall which reflects it. The amplitude of the reflected wave is 50 percent of the amplitude of the incident wave. The result can be expressed as a standing wave plus a one-direction traveling wave. Express the amplitude of the maxima of the standing wave part as a number times the amplitude E_x of the incident wave. Then determine the maximum total displacement at one of the maxima of the standing wave, expressed as a number times the amplitude E_x of the incident wave.
3. **Lowest frequency in a closed organ pipe.** At a closed end of a pipe, there will be a displacement node for sound waves traveling through the pipe. At an open end, there will be a displacement antinode (corresponding to a pressure node there). What is the lowest frequency at which a standing wave can be produced in an organ pipe 0.75 meter long, closed at one end and open at the other?

Brief Answers:

1. a. 432 meter/second

   b. A pressure antinode is a point at which the pressure variation (and the wave) has its maximum possible amplitude.

2. $E = 1.0E_x$, $1.5E_x$

3. 113 hertz