PHYSNET

MISN-0-423

ELECTRIC CURRENT

charge
carrier

electric field

ELECTRIC CURRENT

by

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CONTENTS

Title: **Electric Current**

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Input Skills:

1. Vocabulary: mass current, steady state (MISN-0-418); electric potential, potential drop, volt (MISN-0-420).
2. State the principle of conservation of charge (MISN-0-411).

Output Skills (Knowledge):

K1. Vocabulary: ampere, circuit diagram, effective current, electric current, electric resistance, emf, ohm, parallel connection, series connection, two-terminal system.

K2. State the relationship among currents into and out of a region at steady state.

K3. State the relationship among potential drops along all paths between two points in a circuit.

K4. State the characteristic relationship between current and potential drop for a two-terminal system.

K5. Interchangeably describe a circuit: (a) with a picture or a diagram; (b) with various equivalent circuit diagrams; (c) with a word description; (d) in terms of series or parallel connections.

Output Skills (Rule Application):

R1. Given the average motion of the charged particles in a system, determine the the magnitude and sense of the current in the system.

R2. Given the number of moving charged particles per unit length of wire, the average velocity of these particles, and the sign of their charges, determine the magnitude and sense of the current in the wire.

Output Skills (Problem Solving):

S1. Given the currents and potential drops in one part of a circuit, use the two principles of circuit analysis to determine the currents and potential drops in other parts of the circuit.

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MISN-0-423

ELECTRIC CURRENT

Abstract:

Up to now we have discussed only charged particles which are at rest. In the next three units we shall study the more important and interesting situation where charged particles are moving so that they give rise to electric "currents." Such currents are the basis of most electrical devices and instruments used in everyday life in the laboratory.

SECT.

A DEFINITION AND PROPERTIES OF CURRENT

When charged particles are at rest, the particles in any region remain in that region. But when charged particles are moving, some of the particles in a region ordinarily enter or leave this region by moving through its boundary surface. (The particles, and their associated charge, are then said to "flow" through the surface into, or out of, the region.) How then can one describe the *rate* specifying how rapidly charge flows through any surface?

▶ *Definition of current*

Any surface has two sides and thus separates the adjacent space into the two regions on either side of the surface. To distinguish between these two regions, we can specify one of them by choosing a "sense" indicated by a short arrow pointing from the surface into this region. (See Fig. A-1.) *

> * There are thus only two possible senses, depending on which of the two regions one wishes to specify.

Let Q be the charge in the region specified by the sense arrow. During a small enough time dt, this charge then changes by a small amount dQ as a result of charged particles moving through the surface. The rate at which charge passes through the surface can then be described by the ratio dQ/dt which is called the "electric current" (or the "charge current") flowing through the surface. This current, conventionally denoted by the symbol I, is thus defined as follows:

Def. | **Electric current**: $I = \dfrac{dQ}{dt}$ | (A-1)

PROPERTIES OF ELECTRIC CURRENT

▶ *Current is a number*

According to Def. (A-1), the electric current I is a *number* whose magnitude indicates the magnitude of the charge passing through a surface per unit time. This number may be positive, negative, or zero (depending on whether the charge in the region specified by the sense arrow increases, decreases, or remains the same as a result of the charged par-
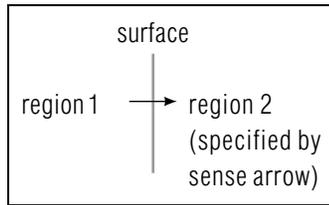
Fig. A-1: Surface separating two regions.

ticles passing through the surface). Note that the electric current is *not* a vector, despite the fact that we have used a sense arrow to specify one of two possible regions. (Indeed, the precise direction of this arrow is irrelevant as long as it points into the region to be specified.)

▶ *Unit of I*

According to Def. (A-1), the unit of current is

$$\text{unit of } I = \frac{\text{coulomb}}{\text{second}} = \text{ampere} \tag{A-2}$$

where the unit "ampere" (abbreviated "A") merely means "coulomb per second." [The name "ampere" commemorates the French physicist Andre Marie Ampere (1775-1836), famous for his studies of the magnetic effects of electric currents.]

▶ *Typical magnitude of I*

The magnitude of the electric current through an ordinary 100 watt light bulb is about 1 ampere. Depending on their magnitudes, electric currents through biological tissues can have pronounced effects. For example, a current of less than 0.001 ampere passing through the human body is not perceptible, but a current 0.01 ampere through the body is painful. A current of 0.1 ampere through the body can cause death by electrocution since it is enough to produce fibrillation of the heart, i.e., disruption of the regular heart rhythm. (Only a small part of this current passes through the heart, but 10^{-5} ampere passing directly through the heart is enough to produce fibrillation.) Finally, currents of several amperes passing through the body produce burns and respiratory paralysis.

PARTICLE MOTION AND SIGN OF CURRENT

▶ *I without net charge*

When some of the charged particles in a material move, they can produce an electric current through a surface although the material has no net charge. This situation is illustrated in Fig. A-2 which shows a metal
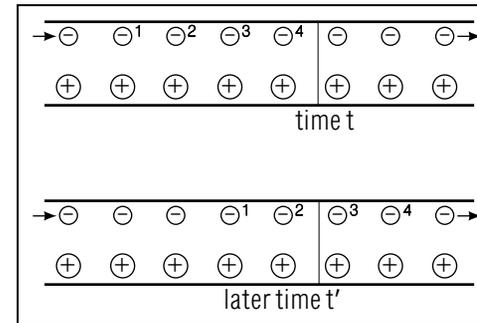
Fig. A-2: Motion of electrons through a metal wire with no net charge. The diagram illustrates very schematically the positions of the electrons and stationary ions at two slightly different times.

wire which is electrically neutral because it contains as many negatively charged electrons as ions with opposite positive charges. Nevertheless, the motion of the electrons through a cross-sectional surface of this wire produces a current through this surface.

▶ *Sign of I*

According to Def. (A-1), the sign of the current $I = dQ/dt$ through a surface, relative to some chosen sense, depends on how the charge Q in the region specified by the sense arrow changes with time as a result of the charged particles moving through the surface. Thus the current I is positive if the charge Q in the specified region increases with time (so that dQ is positive). This can happen either if positively charged particles flow into this region as shown in Fig. A-3a, or if negatively charged particles flow out of this region as shown in Fig. A-3b. Conversely, the current I is negative if the charge Q in the specified region decreases with time (so that dQ is negative). This can happen either if negatively charged particles flow into this region as shown in Fig. A-3c, or if positively charged particles flow out of this region as shown in Fig. A-3d.

To summarize, the current relative to a chosen sense is positive if positively charged particles move along this sense, or if negatively charged particles move opposite to this sense. *

> * We say that particles move (or flow) through a surface along a chosen sense if they pass through the surface toward the region specified by this sense.

Conversely, the current is negative if negatively charged particles move along this sense, or if positively charged particles move opposite to
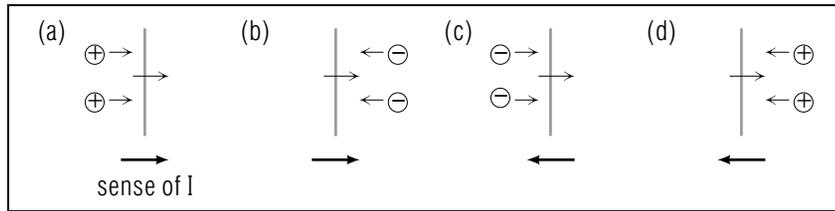
Fig. A-3: Particles with different charges moving in different directions through a surface. The short arrow though the surface indicates a sense chosen so as to specify the region on the right side of the surface. The actual sense of the current is indicated by the arrows at the bottom of the diagram.

this sense. Thus negatively charged particles moving along a chosen sense produce a current of the *same* sign as positively charged particles moving opposite to this sense.

► *Sense of I*

The "sense of the current" through a surface is the particular sense specifying the region in which charge *increases* as a result of the current. *

> * The "sense" of a current is sometimes also called the "direction" of the current, although this wording is confusing because the current is not a vector and thus has no spatial direction, but only one of two possible senses.

If the current along a chosen sense is positive (as in Fig. A-3a and Fig. A-3b), the actual sense of the current is thus the same as this chosen sense. But if the current along a chosen sense is negative (as in Fig. A-3c and Fig. A-3d), the actual sense of the current is opposite to this chosen sense.

Note that the sense of the electric current is the same as the sense of motion of the particles if their charge is positive (as in Fig. A-3a and Fig. A-3d), but is opposite to the sense of motion of these particles if their charge is negative (as in Fig. A-3b and Fig. A-3c).

Example A-1: Electric current in an ionic solution

Figure A-4 illustrates schematically a solution of sodium chloride which contains positively charged sodium (Na^+) ions and negatively charged chlorine (Cl^-) ions. Suppose that the solution is placed in an externally produced electric field $\vec{E}$ pointing to the right. What then is
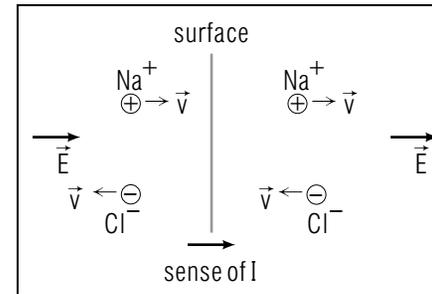
Fig. A-4: External electric field $\vec{E}$ causing motion of Na^+ and Cl^- ions in a solution of sodium chloride.

the current through the indicated surface perpendicular to this field?

The electric field $\vec{E}$ exerts on the positively charged Na^+ ions an electric force to the right and thus causes these ions to move through the surface into the region on the right. Accordingly, these positively charged Na^+ ions moving to the right produce an electric current having a sense toward the right. On the other hand, the electric field $\vec{E}$ exerts on the negatively charged Cl^- ions an electric force to the *left* and thus causes these ions to move through the surface into the region on the left. Accordingly, these negatively charged Cl^- ions moving to the left produce an electric current which has also a sense toward the *right* (since the charge in the right region increases as a result of the motion of negatively charged ions out of this region). Although the external electric field $\vec{E}$ causes the positively and the negatively charged ions to move in opposite directions, the electric currents due to these moving particles have thus the *same* sense (toward the region toward which the electric field points).

Understanding the Definition of Current (Cap. 1d)

A-1 | *Statement and example:* (a) State the definition of current in terms of the usual algebraic symbols for the quantities involved. (b) During a time interval of 4.0 msec (4.0×10^{-3} sec), 2.0×10^{12} sodium ions, each having a charge of 1.6×10^{-19} C, pass into a nerve axon through its membrane. During this time interval, what is the change in the charge of the axon due to ions passing through its membrane? What is the corresponding current through the membrane? *(Answer: 7)*

A-2 | *Properties:* (a) Answer the following questions for charge and for current. What is the usual algebraic symbol for each quantity? Is it a number or a vector? If it is a number, what signs can it have ($+, -, 0$)? What is its SI unit? (b) Is 0.001 ampere, 1 ampere, or 100 ampere a
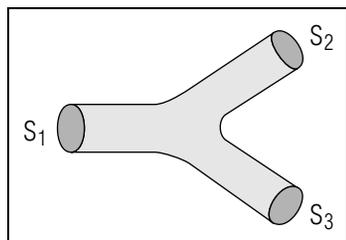
Fig. A-5.

reasonable magnitude for the current flowing through a light bulb in a reading lamp? *(Answer: 4)*

A-3 *Interpretation of dQ:* The region shown in Fig. A-5 has three surfaces through which charged particles move. During a time interval of 0.5 sec, particles with total charge 0.3 C flow into the region through the surface S_1, while particles with total charges of 0.1 C and 0.2 C flow out of the region through the surfaces S_2 and S_3. (a) What are the currents I_1, I_2, and I_3 which flow into this region through each of the surfaces S_1, S_2, and S_3? (b) What is the change dQ in the charge of the region during the 0.5 sec interval described? *(Answer: 1)*

Relating Current Sign and Sense to Particle Motion (Cap. 2a)

A-4 Figure A-6a shows a simplified diagram of a dry-cell battery. Figure A-6b indicates the motion of charged particles across the surfaces of the zinc container and of the carbon rod in its center. Positively charged zinc ions (Zn^{++}) leave the zinc container and become free ions in the water solution. Electrons (e^-) leave the carbon rod (and combine with NH_4^+ ions and with MnO_2 and water molecules to form new compounds). (a) Which of the arrows labeled by 1 and 2 indicates the senses of the currents across the carbon and across the zinc surfaces shown? (b) Relative to a "chosen" sense indicated by the arrow labeled 1, what is the sign of the current across each surface? (c) Relative to a "chosen" sense indicated by the arrow labeled 2, what is the sign of the current across each surface? *(Answer: 10) (Suggestion: [s-11])*

A-5 Figure A-6c indicates the motion of negatively charged electrons in the wire, shown connected to the dry cell in Fig. A-6a. Electrons (remaining in the zinc after the departure of the positively charged zinc ions) move into the wire at its right end. At the left end, electrons leave the wire, moving into the carbon rod, (and ultimately replacing the
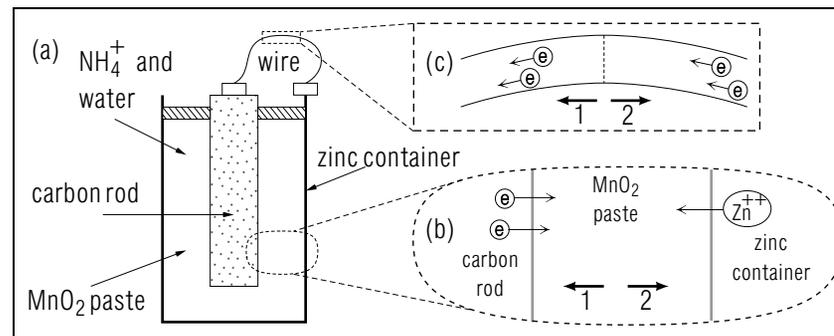
Fig. A-6.

electrons leaving the carbon rod and passing into the solution). (a) Which arrow (1 or 2) indicates the sense of the current through the surface indicated in Fig. A-6c? (b) Is this sense the same or opposite to the sense describing the average motion of the electron? *(Answer: 8)*

A-6 When a current is due to the motion of positively charged particles, is the sense of the current along or opposite to the sense of particle motion? Answer the preceding question for a current due to the motion of negatively charged particles. *(Answer: 5) (Practice: [p-1])*

SECT.
B CURRENT AND MOTION OF CHARGED PARTI- CLES

In the preceding section we examined how the sign, or sense, of a current depends on the motion of the charged particles responsible for this current. Let us now discuss how the *magnitude* of the current is related to the motion of the charged particles.

► *Average velocity*

Consider mobile charged particles in a material (for example, positive or negative ions in a solution, or electrons in a metal). Such particles move around with large speeds in random directions, so that their average velocity along any particular direction is ordinarily zero. (See Fig. B-1a.) But, in the presence of an electric field along some particular direction, the particles tend to move preferentially in the direction of the electric force produced by this field and thus acquire a small average velocity in the direction of this force. (See Fig. B-1b.) Thus electrons in a metal wire acquire an average velocity along the wire if an external device (such as a battery) maintains an electric field inside the wire.

► *Motion perpendicular to surface*

Consider particles, each having a charge q, moving with some average velocity $\vec{v}$ in a direction $\hat{x}$ perpendicular to some plane surface, as indicated in Fig. B-2. (For example, the particles might be electrons in a metal wire and the surface might be a cross section of this wire.) How is the electric current I through such a surface related to the motion of the particles?

► *Characteristics of I*

The sign of the current (relative to the sense indicated by the vector $\hat{x}$ along which the particles move) is positive if the particles are positively charged and is negative if the particles are negatively charged. The *magnitude* of the current (i.e., the magnitude of the rate at which charge passes through the surface) should be larger if there are more particles passing through the surface, if each of these particles has a larger charge, or if these particles move with larger velocity.

CALCULATION OF CURRENT

To make the preceding qualitative comments more quantitative, let us assume for simplicity that every particle has the same charge q and moves
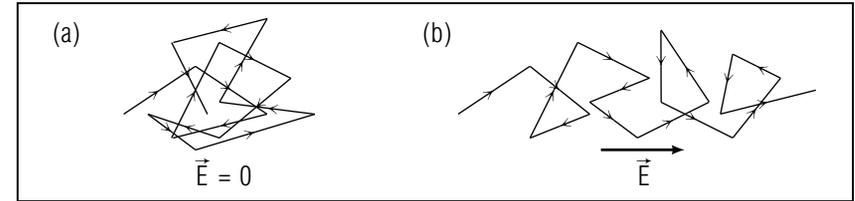
Fig. B-1: Path of a moving charged particle inside a material where it collides repeatedly with the atoms in the material.

with the same velocity $\vec{v}$ (equal to its average velocity) in the direction along $\hat{x}$. As a result of their motion, these particles cross the surface in Fig. B-2 from left to right. During some small time dt, these particles then change the charge in the region on the right side of the surface by some amount dQ, thus producing an electric current $I = dQ/dt$ through the surface (relative to the sense indicated by the vector $\hat{x}$ pointing into the region on the right).

To find this current, consider the N moving charged particles contained at some instant on the left side of the surface in a cylindrical region of small length ℓ and of cross-sectional area equal to that of the surface. (See Fig. B-2.) These particles, moving along $\hat{x}$ with a velocity of magnitude v, require a time $dt = \ell/v$ before all of them pass through the surface. *

> * This is the time after which a particle at the farthest distance ℓ from the surface arrives at the surface as a result of moving with speed v.

Since the total charge of these N particles is Nq, these particles passing through the surface contribute a charge $dQ = Nq$ to the region on the right side of the surface. Accordingly, the electric current I produced by these particles passing through the surface is simply equal to
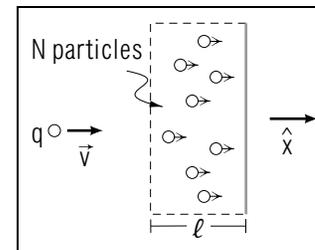

Fig. B-2: Moving charged particles passing through a surface.

$$I = \frac{dQ}{dt} = \frac{Nq}{\left(\dfrac{\ell}{v}\right)} \tag{B-1}$$

Simplifying this fraction by multiplying both its denominator and numerator by v, we then get

$$I = \left(\frac{N}{\ell}\right) qv \tag{B-2}$$

Here N/ℓ is the number of moving charged particles per unit length (in a cylindrical region of area equal to that of the surface).

The relation (B-2) allows us to find the current I through a surface from a knowledge of the number N/ℓ of moving charged particles per unit length, the charge q of each of these particles, and the magnitude v of the average velocity of these particles moving in a direction perpendicular to the surface. Note that the current I increases proportionately to all of these quantities, in accordance with our previous qualitative expectations.

Relating Current Magnitude to Particle Motion (Cap. 2b)

B-1 (a) When a current-carrying metal wire is heated, the number of moving electrons contained in a length ℓ of wire remains (very nearly) fixed. However, their average velocity decreases (due to increased interaction with the more rapidly vibrating atoms of the metal). Does heating a current-carrying wire increase or decrease the magnitude of the current through it? (b) The number of mobile electrons in a semi-conductor increases when more "impurity" atoms are added. If two otherwise identical semi-conductors are connected between the terminals of identical batteries, is the current through the semi-conductor with more impurity atoms larger or smaller in magnitude than the current through the semi-conductor with fewer such atoms? *(Answer: 12) (Suggestion: [p-2])*

Illustration: Speed of Electrons In Wires

B-2 Standard "lamp cord" contains about 7.0×10^{22} atoms of copper in each meter of its length. Each atom of copper has 1 mobile electron with a charge of 1.6×10^{-19} coulomb. If a current of magnitude 2.0 ampere flows through such a wire, what is the magnitude of the average velocity of the moving electrons in the wire? *(Answer: 2)*

SECT.

C STEADY STATE CURRENTS

▶ *Definition of "steady state" flow*

Suppose that a metal wire, or some other system, is connected to some external device (such as a battery) which can maintain an electric current through the wire. Immediately after the wire is connected, the electrons rearrange themselves throughout the wire so that the motion of these electrons is quite complex. But, after a very short time, the motion of the electrons usually settles down to a simple "steady-state" situation characterized as follows:

Def. | **Steady state**: A situation in which the total charge in any region remains unchanged. (C-1)

Analogous steady-state situations occur commonly in the flow of liquids (as discussed in text section B of Unit 418). For example, when a hose is first connected to a water faucet, the flow of the water through the hose is initially quite complex as the water gradually fills the hose. But, after a short time, the hose becomes filled and the situation settles down to a steady state where the water is flowing through the hose while the amount of water in any region of the hose remains unchanged.

▶ *Currents in steady state*

Because of the conservation of charge, the total charge in any region can only change because of charges passing through its boundary surface. In a steady state, where the total charge in any region remains unchanged, the total electric current I_{in} flowing into the region (through various parts of its boundary) must then be equal to the electric current I_{out} flowing out of the region (through various other parts of its boundary). This fundamental condition relating the currents in a steady state can thus be summarized:

$$\boxed{I_{in} = I_{out}.} \tag{C-2}$$

This relation can be easily visualized by thinking of the analogous flow of a liquid in a steady state (as discussed in text section B of Unit 418). During any time, the flow of liquid *into* any region must be equal to the amount of fluid flowing *out* of this region.
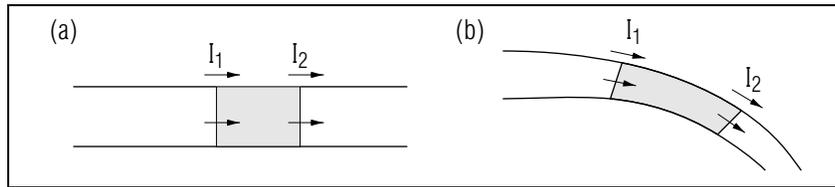
Fig. C-1: Currents flowing through wire.

CURRENT THROUGH A WIRE

Consider any metal wire (not necessarily of uniform cross-section), such as the straight wire in Fig. C-1a or the curved wire in Fig. C-1b. In a steady state, what can we say about the current at any point in the wire (i.e., about the current through a cross-sectional surface of the wire at any such point)?

► *Equality of currents*

Let us choose senses at various points along the wire in such a way that the tip of the arrow indicating one sense always points toward the tail of the arrow indicating the next sense. (See Fig. C-1.) Let us denote by I_1 and I_2 the currents, relative to these chosen senses, through the cross-sectional surfaces at any two points of the wire. If these currents are positive, the current into the region between the cross-sectional surfaces in Fig. C-1 is then I_1, while the current out of this region is I_2. In a steady state, the relation $I_{in} = I_{out}$ thus implies simply that *

$$I_1 = I_2 \qquad (C\text{-}3)$$

---

* If the currents I_1 and I_2 in the preceding discussion were negative, I_1 would be the current *out* of the region and I_2 would be the current *into* the region. Thus the equality $I_1 = I_2$ of Eq. (C-3) would remain valid.

In a *steady state*, the *same* current flows thus through any cross-sectional surface of the wire. Accordingly we can simply talk about the current I flowing through the wire, without needing to specify any particular cross-section of the wire.

► *Unconnected wire*

If one end of a wire is unconnected to anything else, the current flowing through this end of the wire must be zero. In a steady state, the
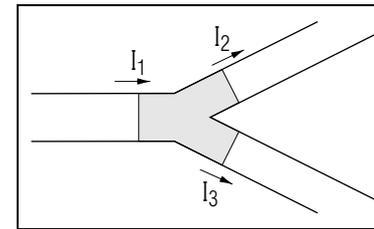
Fig. C-2: Currents in wires joined together. The junction region is shown shaded.

current through any other cross-section of the wire must then, by Eq. (C-3), also be zero (irrespective of what the other end of the wire might be connected to).

► *Current and particle flow*

Suppose that the current I in a wire flows in the sense indicated by the arrows in Fig. C-1. As discussed in Sec. A, this current can be either the result of positively charged particles moving through the wire along this sense, or of negatively charged particles moving through the wire opposite to this sense. But, irrespective of the actual situation, it is simplest to imagine that the current results from the flow of positively charged particles, since the sense of the electric current is then the same as that of the motion of the particles.

CURRENTS AT A JUNCTION

Consider several wires (such as the three wires in Fig. C-2) which meet at a junction. In a steady state, how are the currents in these wires related?

Let I_1, I_2, and I_3 denote the currents through these wires relative to senses chosen as indicated by the arrows in Fig. C-2. In a steady state, the total current I_{in} into the junction region must be equal to the total current I_{out} out of this region. But in Fig. C-2 the current into the junction region is $I_{in} = I_1$ while the current out of this region is $I_{out} = I_2 + I_3$. Hence the relation $I_{in} = I_{out}$ implies that

$$I_1 = I_2 + I_3 \qquad (C\text{-}4)$$

This relation allows us to find the current in one of the wires from a knowledge of the currents in the other wires. *

* Any of the currents in Eq. (C-4) might be negative. This
merely means that the actual sense of such a current is oppo-
site to the chosen sense indicated in Fig. C-2.

The situation in Fig. C-2 is again analogous to that of the steady
flow of a liquid through several pipes joined together. During anytime,
the amount of liquid flowing into the junction region through one pipe is
simply equal to the amount of fluid flowing out of this region through the
other pipes.

REMARK ON ALTERNATING CURRENTS

As will be discussed more fully in text section D of Unit 425, the
currents supplied to our homes through wall outlets are "alternating,"
i.e., they vary repetitively in time (at a rate of 60 repetitions per second
in the United States) so as to be as often positive as negative the mag-
nitude of such an alternating current can be described by an effective
value (which is a fixed fraction of its maximum value). Such alternating
currents commonly also flow in a steady state where the total charge in
any region remains unchanged. Hence the relation $I_{\text{in}} = I_{\text{out}}$ also holds
for alternating currents at any instant, or for their effective values.

Understanding $I_{\text{In}} = I_{\text{out}}$ (Cap. 1b)

C-1 *Example:* Figure C-3 shows a floor plan of a kitchen in which
current flows through the stove, refrigerator, and through a wall
outlet to which is connected a toaster and coffee maker. Arrows and
numbers in Fig. C-3 indicate the magnitudes and senses of the currents
flowing through the kitchen walls at one instant of time. What are the
currents I_{in} and I_{out} for this kitchen? *(Answer: 9)*

C-2 *Applicability:* (a) The initially uncharged plates of a capacitor are
connected to the terminals of a battery. Consider *one* capacitor
plate during the time this plate is acquiring a positive charge from the
battery. For this plate, I_{in} has some positive value, but $I_{\text{out}} = 0$. Why
does the relation $I_{\text{in}} = I_{\text{out}}$ *not* apply to this situation? (b) Now consider
a later time when the same plate is fully charged. Now no current flows
in or out of the plate so that $I_{\text{in}} = I_{\text{out}} = 0$. Why is $I_{\text{in}} = I_{\text{out}}$ applicable
in this situation, although it was not applicable to the situation described
in part (a)? *(Answer: 11) (Suggestion: [s-8]) (Practice: [p-3])*

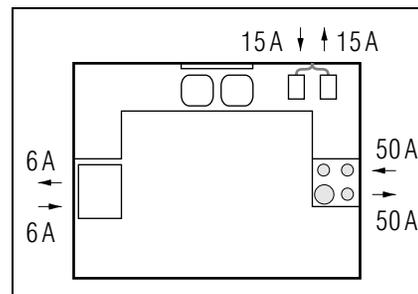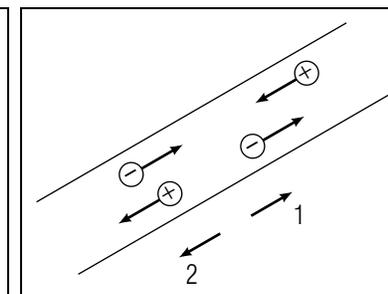Fig. C-3.   Fig. C-4.

Relating Current Sign and Sense to Particle Motion (Cap. 2a)

C-3 Figure C-4 shows the velocities of positively charged neon ions
and negatively charged electrons along the long tube of a neon
light. (a) Which arrow (1 or 2) indicates the sense of the current due
to the electrons and due to the ions? (b) Relative to a chosen sense
indicated by arrow 1, what is the sign of each of these currents? *(Answer:
6) (Suggestion: [s-5])*

Using $I_{\text{In}} = I_{\text{out}}$ to Relate Currents (Cap. 3)

Now: Go to tutorial section C.

C-4 *Recognizing equal magnitudes:* The diagrams in Fig. C-5 each
show a set of connected wires with arrows indicating the senses of
the steady currents through them. (a) In Fig. C-5a, currents flow through
surfaces at the points A, B, C, D, and E. Through which of these sur-
faces must the current be equal in magnitude to the current through the
surface at A? (b) In Fig. C-5b, through which of the light bulbs A, B,
C, and D must the current be equal in magnitude to the current through
the battery? *(Answer: 3) (Suggestion: [c-2])*

C-5 *Relating senses:* Figure C-6 shows several sets of connected wires.
In each set, single arrows indicate the known sense of the current
in one or more wires. A pair of arrows (labeled 1 and 2) indicate the two
possible senses of an unknown current in another wire. From each pair of
arrows, choose the one which indicates the sense of the current (or state
that this current could have either sense). *(Answer: 15) (Suggestion:
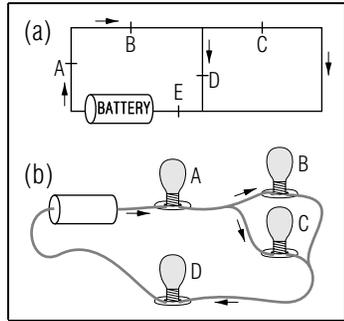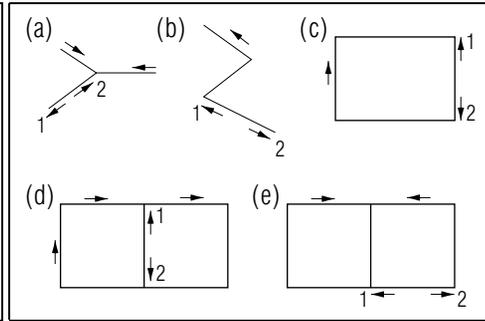[c-3])*

Fig. C-5.

Fig. C-6.

C-6 *Relating magnitudes and senses:* Figure C-7 shows a diagram of the currents flowing in a house trailer. Find the magnitude and sense of the current I_l through the living room lights by applying $I_{\text{in}} = I_{\text{out}}$ to the indicated region. *(Answer: 18)*

C-7 *Relating magnitudes:* Figure C-8 shows the senses of various currents flowing in connected wires in a kitchen. If the current into (and out of) the refrigerator, toaster, and coffee maker have the magnitudes 6.6 A, 8.3 A, and 12.2 A, what is the magnitude of the current I flowing out of the kitchen? *(Answer: 13) (Suggestion: [s-7]) (Practice: [p-4])*

C-8 *Switches and zero currents:* A switch is simply a device which can be either "closed" so as to connect two wires or "open" so as to disconnect these wires from each other. In Fig. C-8, the wire through which current flows away from the coffee maker has a switch labeled by S. Suppose this switch is open. Which of these currents are then zero? (a) Current out of the coffee maker. (b) Current into the coffee maker. (c) Current into the toaster. *(Answer: 16) (Suggestion: [s-6])*
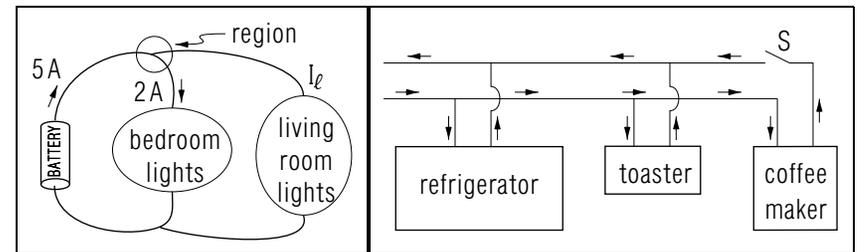

Fig. C-7.

Fig. C-8.

SECT.

D TWO-TERMINAL SYSTEMS AND CIRCUITS

► *Two-terminal systems*

In many important practical situations, electric currents flow through various systems such as lamps, tape-recorders, biological organisms, and so forth. To discuss such systems, we shall introduce this definition:

Def.	**Two-terminal system**: Any system with two points (called its "terminals") such that electric current can flow into the system through a small surface at one terminal and can flow out of the system through a small surface at the other terminal.	(D-1)

Such a two-terminal system is shown schematically in Fig. D-1. Note that a two-terminal system can be of any degree of complexity. For example, such a system might simply be a copper wire whose two ends are the terminals of the system. Or it might be a vacuum cleaner whose two terminals are the two prongs on the plug. Or it might be a man whose two terminals are the thumbs of his hands.

► *Description of a system*

The electric properties of a two-terminal system can be described quite usefully without needing to know any details about the internal structure of the system. Thus we may describe such a system merely in terms of quantities which can be measured at its terminals. These quantities are the electric currents flowing through these terminals and the potentials at these terminals. (As we shall see, a knowledge of how these quantities are related can provide very useful information about two-terminal systems and their interaction.)

► *Steady state*

In a steady state, the total charge in a two-terminal system remains unchanged. Hence the electric current flowing into the system through one terminal must then be equal to the current flowing out of the system through its other terminal. We shall call this current the "current through the system."

► *Non-dissipative systems*

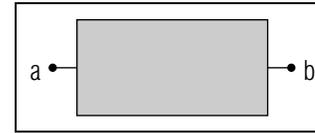In some special two-terminal cases (e.g., in a vacuum or in a super-

Fig. D-1: Two terminal system with terminals a and b.

conductor) charged particles coast along with constant velocity if there is no electric field present. That is because there is no mechanism for the dissipation of their kinetic energies. The charged particles can be made to accelerate (or decelerate) by applying a field.

► *Dissipative systems*

Ordinarily, however, the charged particles moving through a two-terminal system interact (or collide) with the many atoms in the materials within the system. As a result of such interactions, the energy of the moving charged particles is "dissipated," i.e., is converted into the random internal energy of all the atomic particles in the material. (Such a two-terminal system is therefore called "dissipative.") Hence a steady current can flow through such a system only if the energy lost by the particles to random internal energy is resupplied from other sources. For example, the energy lost to random internal energy may be supplied at the expense of a decrease in the coulomb electric potential energy of the charged particles moving through the system.

► *Characteristic relation*

The decrease in the coulomb potential energy of the charged particles moving through a system is simply related to the potential drop V between its terminals. Hence we expect that the steady current I flowing through the system should be related to the potential drop V "across" the system (i.e., from one terminal to the other). The exact relation between I and V is characteristic of the nature of the particular system under consideration. We shall discuss in Sec. F how I is related to V in the case of the most common two-terminal systems.

CIRCUITS

Two-terminal systems can be connected together so that the current flowing out of a terminal of one system can flow into a terminal of another system. Any collection of two-terminal systems connected together is called an electric "circuit" (or "network").
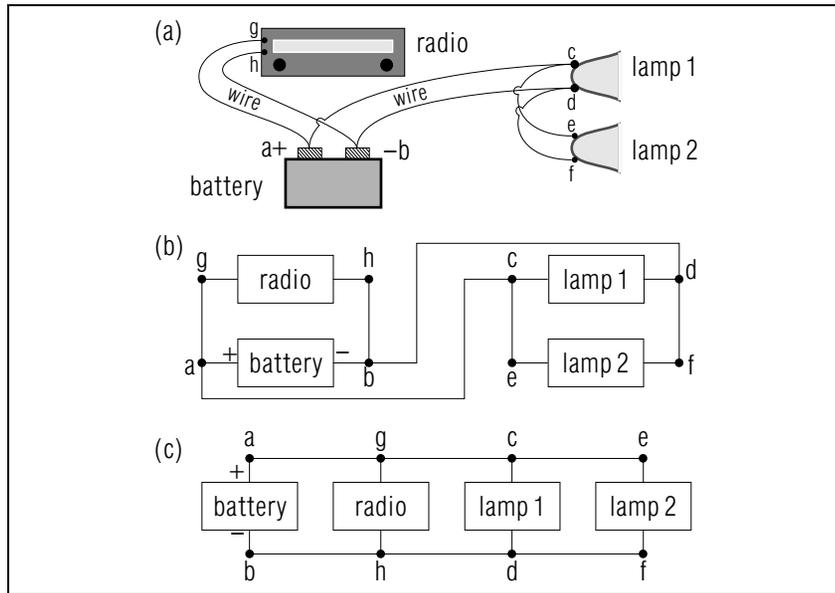
Fig. D-2: Circuit consisting of a storage battery, two head-lamps and a radio: (a) physical arrangement; (b) and (c), equivalent circuit diagrams representing this circuit. (The connecting wires in the actual circuit are assumed short enough so the potential drop along each of them is negligible.)

► *Circuit diagram*

A circuit diagram (such as the one illustrated in Fig. D-2b) is a drawing representing symbolically how the various two-terminal systems in a circuit are connected together. In such a diagram, a line connecting any two terminals is used to indicate that current can flow between these terminals and that these terminals have the same potential. (For example, such a connecting line might represent the fact that the actual terminals touch each other or are connected by a sufficiently short metal wire.)

Note that a circuit diagram indicates only the *connections* between terminals, *irrespective* of the actual positions of the systems in the diagram. For example, Fig. D-2a shows a drawing of an automobile storage battery connected by insulated wires to two headlamps and a car radio. *

> * For any battery, the terminal at the higher potential is con-ventionally indicated by a + sign and the terminal at the lower potential by a − sign.

This physical situation is represented by the two equivalent circuit diagrams shown in Fig. D-2b and Fig. D-2c. Although these two diagrams look different, they are completely equivalent because they show precisely the same terminals connected to each other.

SPECIAL CIRCUIT CONNECTIONS

Two or more two-terminal systems can be connected together in some special simple ways so that the resulting circuit is another two-terminal system.

► *Series connection*

Two or more systems can be connected like a chain, as shown in Fig. D-3a, so that one terminal of each system is connected to one terminal of the next system. The systems are then said to be connected "in series." One terminal of the first system and one terminal of the last system are left unconnected (i.e., they are the ends of the chain) and are the terminals of the composite two-terminal system consisting of the individual systems connected in series.

► *Parallel connection*

Two or more systems can also be connected, as shown in Fig. D-3b, by connecting together one terminal of *each* system and also connecting together the other terminal of *each* system. The systems are then said to be connected "in parallel." Note that an electric current traveling from g to h in the diagram will split in two at point e, part of the current traveling the route $eabf$ and the other part traveling the "parallel" route $ecdf$.

Knowing About Two-Terminal Systems

D-1 Which of these systems (a and b) in Fig. D-4 are two-terminal systems? Suppose that the tape-recorder in Fig. D-4b is set on "play" so that no current flows through the wires to the microphone. Is this tape recorder then a two-terminal system? *(Answer: 22)*
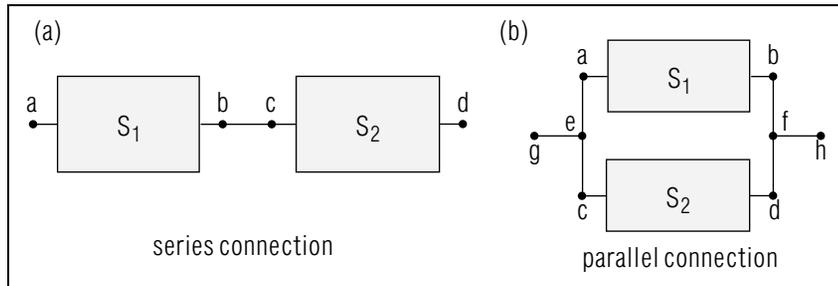
Fig. D-3: Special connections of two two-terminal systems S_1 and S_2.

Knowing About Current and Work Per Unit Charge

D-2   An electron moves from a battery into a wire at a terminal A. It then moves with constant average velocity along the wire (a dissipative two-terminal system) until it reaches the point B. (a) As the electron moves from A to B, does its coulomb potential energy increase, decrease, or remain the same? (b) Does its average kinetic energy increase, decrease or remain the same? (c) Describe any transfer of energy from the electron to any other system. *(Answer: 19) Now: Go to tutorial section D.*

Relating Descriptions of Circuits (Cap. 4)

D-3   *Recognizing points with equal potentials:* Which of the labeled points in Fig. D-5 must have the same potential as the positive terminal of the battery? Which must have the same potential as the
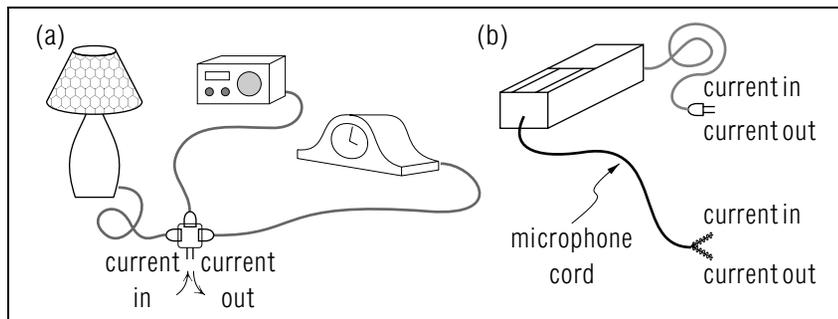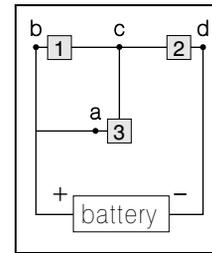

Fig. D-4.

Fig. D-5.

negative terminal of the battery? *(Answer: 17) (Suggestion: [s-3])*

D-4   *Recognizing equivalent circuit diagrams:* Which of the diagrams (b) and (c) in Fig. D-6 is/are equivalent to the diagram (a)? *(Answer: 14) ([d-2], [d-3])*

    Throughout this unit, we shall assume that terminals connected by metal wires have the same potential. [This assumption is very nearly correct for short wires. We shall discuss it further in Unit 424.]

D-5   *Relating pictures and drawings of circuits:* Figure D-7 shows three labeled light bulbs connected to a battery. Which circuit diagram (or diagrams) in Fig. D-6 describes this circuit? *(Answer: 23) ([d-3], [d-4])*

D-6   *Making circuit diagrams:* Figure D-8 shows parts of two strands of Christmas-tree lights. Draw a circuit diagram for each string of lights. Use a labeled square to indicate each light bulb. *(Answer: 26) (Suggestion: [s-14])*

D-7   *Parallel and series connections:* (a) Two batteries in a flashlight are connected in series. Use a rectangle to represent each battery and draw a diagram showing how they are connected. (b) A fluorescent light bulb and an incandescent light bulb are connected in parallel in a reading lamp. Use a rectangle to represent each bulb and draw a diagram showing how they are connected. *(Answer: 20) (Suggestion: [d-5])*

D-8   *Parallel and series connections:* In the diagram shown in Fig. D-5, which of the following pairs of systems are connected in series, which are connected in parallel, and which are connected neither in parallel nor in series: (1 and 3), (2 and 3), (battery and 2)? *(Answer: 30) (Suggestion: [d-5]) More practice for this Capability: [p-5], [p-6], [p-7]*
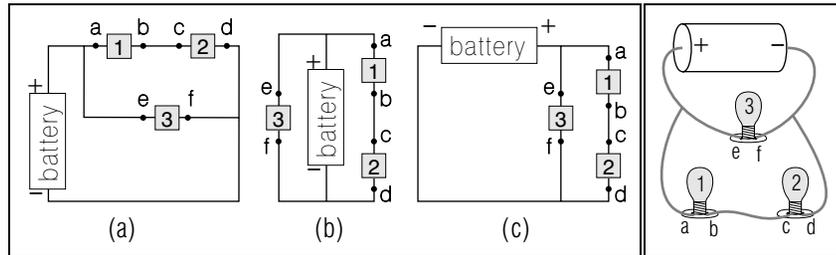
Fig. D-6.

Fig. D-7.

Fig. D-8.

SECT.

E PRINCIPLES OF CIRCUIT ANALYSIS

Let us now summarize the general principles which allow us to relate the currents through various two-terminal systems connected in a circuit, and also to relate the potential differences between the terminals of these systems. By combining these principles with a knowledge of the relation between the current and potential difference for *each* two-terminal system, we should then be able to analyze any circuit to find unknown currents or potentials on the basis of limited information.

▶ *Relation between currents*

Unless specifically stated otherwise, we shall consider circuits in a steady state. If we consider any junction where two or more terminals are connected together, we then know that the charge in this junction region must remain unchanged. By Eq. (C-2), the total current I_{in} flowing into the junction region must then be equal to the total current I_{out} flowing out of this region. Thus

$$\boxed{\text{at any junction, } I_{\text{in}} = I_{\text{out}}} \qquad \text{(E-1)}$$

▶ *Relation between potentials*

The potential at every point has some unique value. Hence the potential drop V_{ab} from any point a to any other point b has also some unique value. But this potential drop is equal to sum of the potential drops along successive segments of *any* path from a to b. * To summarize:

> From any point a to any other point b, the potential drop V_{ab} is the sum of the potential drops along any path from a to b. (E-2)

> * This statement is equivalent to saying that the work done on a charged particle moving along any path from a to b is just the sum of the works done along successive segments of this path.
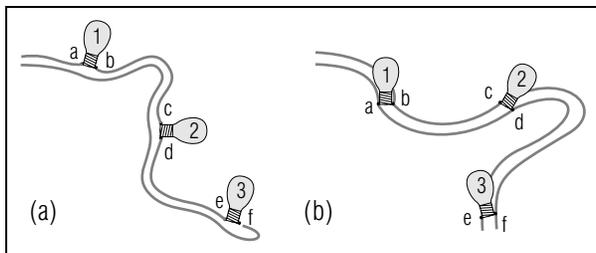
In particular, this conclusion implies that the sum of the potential drops calculated along any one of several alternate paths from a to b must be the *same*, equal to the potential drop V_{ab}. *

> * This statement is a consequence of the fact that the work done by coulomb forces on a charged particle moving between two points is independent of the path.

ANALOGY TO ELEVATION

The preceding conclusions are analogous to facts familiar in everyday life. Every point in a terrain has some unique elevation above sea level. In going on a hike from a point a to another point b, the drop in elevation is then simply the sum of the drops in elevation along successive portions of *any* hiking path from a to b. Correspondingly, the sum of the drops in elevation along alternate hiking paths from a to b must be the *same*, equal to the drop in elevation from a to b.

APPLICABILITY OF ANALOGOUS PRINCIPLES TO FLUIDS

Principles analogous to Eq. (E-1) and Rule (E-2) apply in discussing the steady flow of fluids (such as that of water through the pipes of a plumbing system, or blood through blood vessels). The principle Eq. (E-1) then relates the fluid (or mass) currents into and out of any region. The other relevant quantity is the pressure (instead of the electric potential). Since the pressure has a unique value at every point, the principle corresponding to Rule (E-2) is that the pressure drop from any point a to any other point b is the sum of the pressure drops along any path from a to b.

Example E-1: Finding unknown quantities in a circuit

Figure E-1 shows a circuit diagram representing the storage battery of a car connected through a fuse to a radio and a lamp. The battery maintains between its terminals a and b a potential drop $V_{ab} = 6.2$ volt. The current I_B flowing through the battery is 3.5 ampere in the indicated sense. The measured potential drop V_{cd} from terminal c to terminal d of the lamp is $V_{cd} = 6.1$ volt and the current I_L through the lamp is 3.2 ampere in the indicated sense. What then is the current through the fuse and the potential drop between the terminals of the fuse? Also, what is the current through the radio and the potential drop between the terminals of the radio?

In the steady state, the current I_B flowing out of the battery into the wire near a must be equal to the current flowing out of the wire
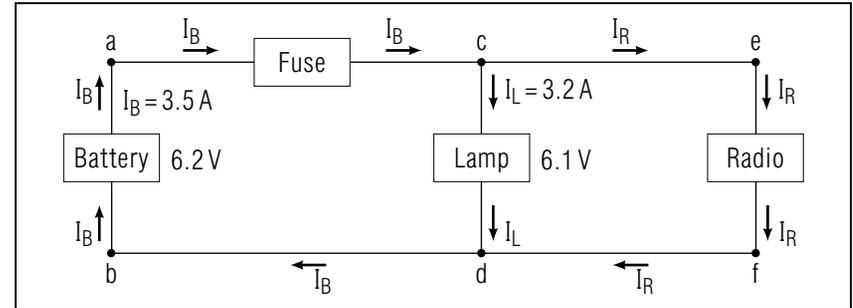

Fig. E-1: Circuit consisting of a storage battery connected to a lamp and a radio.

into the fuse. Thus the current through the fuse is also $I_B = 3.5$ ampere in the sense indicated in the diagram. This current I_B then flows out of the fuse into the junction point c. The current then flows out of this junction point along two possible paths, one part I_L of this current flowing through the lamp and another part I_R of the current flowing through the radio. In a steady state, the currents into and out of the junction c are related so that $I_B = I_L + I_R$. Hence the current through the radio is $I_R = I_B - I_L = 3.5$ ampere $- 3.2$ ampere $= 0.3$ ampere. (Note that the currents I_L and I_R recombine at the next junction point d so that the current flowing along the path from d to b is $I_L + I_R = I_B$, which is properly just the current flowing back through the battery.)

Let us now look at the potential drops. Considering the points a and b, the potential drop V_{ab} along the path through the battery is known to be 6.2 volt. But this potential drop must be the same as that along the alternate path $acdb$ through the fuse, the lamp, and the connecting line db (along which the potential drop is zero). The potential drop along this path is then $V_{ab} = V_{ac} + V_{cd} + 0$. Hence 6.2 volt $= V_{ac} + 6.1$ volt so that the potential drop V_{ac} from terminal a to terminal c of the fuse must be equal to $V_{ac} = 0.1$ volt.

Note also that the known potential drop $V_{cd} = 6.1$ volt along the path through the lamp must be the same as along the path c, e, f, d through the radio. But the potential drop along this alternate path is just the potential drop V_{ef} through the radio (since the potential drops along the connecting paths from c to e, and from f to d, are zero). Thus $V_{cd} = V_{ef}$ so that the potential drop between the terminals e and f of the radio is equal to the potential drop $V_{cd} = 6.1$ volt between the terminals of the lamp.
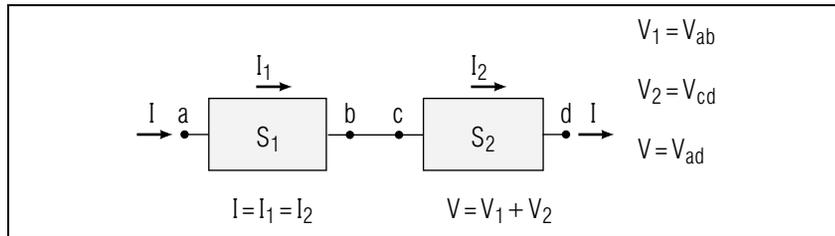
Fig. E-2: Two-terminal systems S_1 and S_2 connected in series.

SERIES AND PARALLEL CONNECTIONS

The principles of circuit analysis have immediate implications for systems connected in series or parallel (as defined at the end of Sec. D.)

► *Series connection*

Consider two systems connected in series, as shown in Fig. E-2. In a steady state, the current flowing out of one system must be equal to the current flowing into the next system. Thus the *current* through each system is the *same*, equal to the current I which flows through the composite two-terminal system. On the other hand, the total potential drop V across the composite system (i.e., the potential drop from terminal a to terminal d in Fig. E-2) is equal to the *sum* of the potential drops across each of the individual systems.

► *Parallel connection*

Consider now two systems connected in parallel, as shown in Fig. E-3. Since the corresponding terminals of these systems are connected together, they must have the same potential. Hence the potential drop V across the composite system (i.e., from terminal e to terminal f in Fig. E-3) must be the *same* as the potential drop across each individual system. On the other hand, the current I which flows into the composite system flows partially through each of the connected systems. In a steady state, the current I flowing into the system must then be equal to the *sum* of the currents flowing through the individual systems. (Thus, in Fig. E-3, the current I flowing into the junction at the point e must be equal to the sum of the currents I_1 and I_2 flowing out of this junction through the individual systems.)
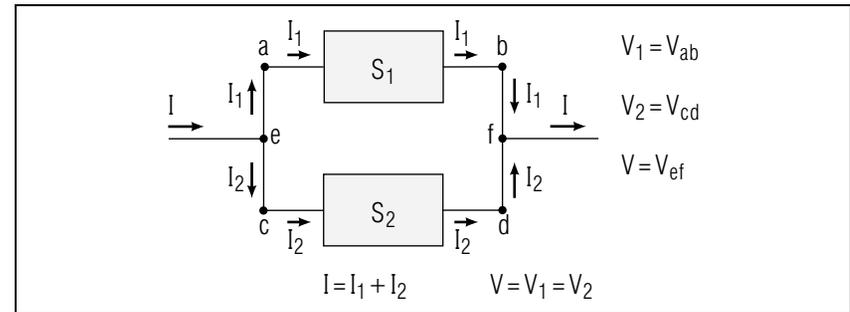
Fig. E-3: Two-terminal systems S_1 and S_2 connected in parallel.

Relating Currents and Potentials In a Circuit (Cap. 3)

Now: Go to tutorial section E.

E-1 A current of 2.0 ampere flows into a two-terminal system consisting of two light-bulbs connected in parallel . The potential drop across one light bulb is 100 volt, and the current through it is 1.4 ampere. (a) Draw a circuit diagram showing the two light bulbs and the senses of the currents flowing through them. (b) Apply $I_{in} = I_{out}$ to find the magnitude of the current flowing through the second light bulb. (c) Apply $V_{ab} =$ sum of potential drops along any path to find the potential drop across this second light bulb. *(Answer: 27) ([e-1], [e-2])*

E-2 A current of 2.0 ampere flows into one terminal of a two-terminal system consisting of two light bulbs connected in series. The potential drop V across the combined system is 100 volt. The potential drop V across the first light bulb is 70 volt. What is the potential drop V across the second light bulb, and what is the magnitude of the current through this bulb? *(Answer: 21) (Suggestion: [s-13])*

E-3 Figure E-4 shows the circuit diagram of an emergency electrical system for a summer cabin often left without outside electrical power. The arrows indicate the sense of the current in each part of the circuit. (a) If each battery is an automobile battery which produces a constant potential difference of 6.0 volt between its terminals, what is the potential drop from a to b? (b) Suppose that the current through the radio is 0.5 ampere, the current through the battery is 1.4 ampere, and the water pump is turned off so that no current flows through it. What then is the current through each of the two light bulbs? *(Answer: 24)*
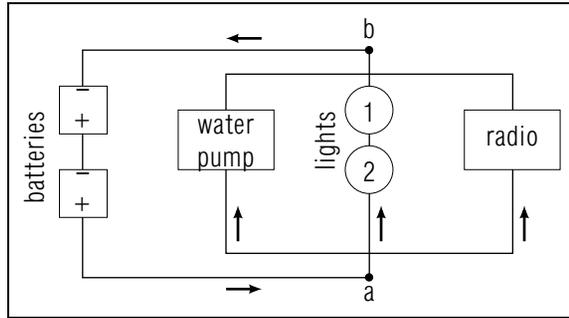
Fig. E-4.

(Practice: [p-8])

Knowing About Series and Parallel Connections

E-4 Two two-terminal systems (A and B) can be connected either in series or in parallel. Which of the following statements describes a system composed of A and B connected in series, and which describes a system composed of A and B connected in parallel? (a) Current through A equals current through B. (b) Potential drop across A equals potential drop across B. (c) Current into combined system equals sum of currents through A and through B. (d) Potential drop across entire system equals sum of potential drop across A and across B. *(Answer: 33)*

SECT.

F CHARACTERISTICS OF A TWO-TERMINAL SYSTEM

► *Current and energy dissipation*

As discussed in Sec. D, charged particles moving through a dissipative two-terminal system interact with the atoms in the system. As a result, these charged particles lose their energy so as to increase the random internal energy of all the particles in the system. Hence a steady electric current through the system can only be maintained if the energy thus transformed into random internal energy is equal to the non-random energy supplied to (or work done on) the moving charged particles. *

> * Changes in the kinetic energy of the moving charged particles are so small as to contribute negligibly to the random internal energy of the system.

► *Relating current and work*

Consider a two-terminal system with terminals a and b. (See Fig. F-1.) To maintain a steady current I through this system from a to b, some positive non-random work per unit charge w must be done on the charged particles (moving through the system from a to b) in order to resupply the energy lost by these particles to the random internal energy of the system. On the other hand, the steady current through the system is zero if no work is done to resupply energy lost to random internal energy. Thus $I \neq 0$ only if $w \neq 0$, and $I = 0$ if $w = 0$. If the current I is sufficiently small, this current must then be simply proportional to the work w. *

> * The reason is that a graph showing the relation between I and w must be nearly a straight line in a sufficiently small region near the point where $I = 0$ and $w = 0$.

Accordingly, we can write

$$I = Gw = \frac{1}{R}w, \text{ or } RI = w \tag{F-1}$$

where the quantity G (or $R = 1/G$) is some constant, independent of I or w, which characterizes the particular two-terminal system. The quantity R is called the "resistance" of the system and its reciprocal $G = 1/R$ is called the "conductance" of the system.
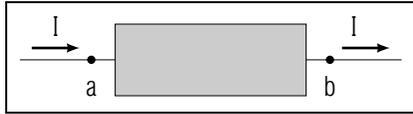
Fig. F-1: Steady state current I flowing through a two-terminal system.

► *Kinds of work done*

What is the non-random work per unit charge w, done on a charged particle moving through the system from a to b? Some of this work is the *coulomb* electric work per unit charge. This work is simply equal to the potential drop $V = V_{ab} = V_a - V_b$ from a to b (i.e., to the drop in coulomb potential energy per unit charge from a to b). But in complex systems *non-coulomb* work may also be done. (For example, in a battery chemical reactions can supply chemical energy, and thus do chemical work, when charged particles move from one terminal to the other.) Such non-coulomb work is called "emf" in accordance with this definition:*

> * The term "emf" is an abbreviation (pronounced as the separate letters e,m,f) for the phrase "electro-motive force." The full words are today almost never used and are misleading, erroneously using "force" instead of "work per unit charge."

Def. | **Emf**: The emf $\mathcal{E} = \mathcal{E}_{ab}$ from a point a to a point b along some specified path, is the non-coulomb work per unit charge done on a charged particle moving along this path. | (F-2)

The non-coulomb work per unit charge done on a charged particle moving through the system from a to b is thus called the emf $\mathcal{E} = \mathcal{E}_{ab}$ of the system from a to b. (This emf is ordinarily independent of the current I or potential drop V through the system.) Hence the total non-random work per unit charge w, done on a charged particle moving through the system from a to b is simply the sum of the coulomb and non-coulomb forces per unit charge, i.e.,

$$w = V + \mathcal{E} \tag{F-3}$$

► *Relation between I and V*

Accordingly, the relation (F-1) between current and non-random work can be written more explicitly as

$$\boxed{RI = V + \mathcal{E}.} \tag{F-4}$$

This "characteristic relation" between the current I through the system and the potential drop V across its terminals is valid as long as I is not too large. We shall use this relation throughout the next two units to discuss various kinds of practically useful two-terminal systems.

DISCUSSION

The relation (F-4) shows how the electric current through a two-terminal system is due both to the coulomb work done on the moving charged particles because of the potential drop between the terminals, and also to the non-coulomb work done on these particles. [Analogously, the fluid current of a liquid through a horizontal pipe is due both to the work done on the moving liquid because of the pressure drop between the ends of the pipe, and also to other kinds of work done by devices such as pumps.]

► *Utility of the relation*

The relation (F-4) is valid for *any* dissipative two-terminal system as long as the current through it is not too large. (In practice, this relation is usually valid for all currents of interest.) Note that Eq. (F-4) describes explicitly how the steady current I through any such two-terminal system is related to the potential drop V between the terminals of the system. This relation involves two constants characterizing a particular system, namely the electric resistance R of this system and the emf $\mathcal{E}$ of this system.

► *Meaning of R*

By Eq. (F-1) or Eq. (F-4), the resistance R of a system is a numerical quantity describing the relation between the current I through the system and the work $w = V + \mathcal{E}$ done per unit charge. Thus the resistance is defined as follows (for currents which are sufficiently small):

Def. | **Resistance**: $R = \dfrac{w}{I} = \dfrac{V + \mathcal{E}}{I}$ | (F-5)

If a given amount of work per unit charge w produces a large current, the resistance R of the system is thus small (and the conductance $1/R$ of the system is correspondingly large). On the other hand, if the same given amount of work per unit charge w produces only a small current, the resistance R of the system is thus large (and the conductance $1/R$ of the system is correspondingly small). These comments suggest why the quantity R describes the resistance to current flow.

► *Unit of R*

The work per unit charge w has the same unit as the potential drop, i.e., volt. According to Def. (F-5) the unit of R is then;

$$\text{unit of } R = \frac{\text{volt}}{\text{ampere}} = \text{ohm}. \qquad (\text{F-6})$$

Here the new unit "ohm" [abbreviated by the Greek letter "Ω" (omega)] is just a convenient abbreviation for the combination volt/ampere. This unit is named in honor of Georg S. Ohm (1789-1854), the German physicist who studied steady electric currents in wires.

► *Sign of R*

To maintain a steady positive current flowing through a two-terminal system from terminal a to terminal b, the work per unit charge w, done on a particle moving from a to b, must be positive (or zero in the special case where the dissipation of energy into random internal energy is negligibly small). For only then will the work done on any positive amount of charge, passing through the system from a to b, be properly positive so as to increase the random internal energy of the system. Hence we conclude that the resistance $R = w/I$ of a system must always be positive or zero.

Illustration: Work Done on Particles In a Battery

| F-1 | Figure F-2 shows a circuit, including a battery through which flows a current I. Consider a positively charged particle moving through the battery from its negatively charged terminal b to its positively charged terminal a. (a) Is the coulomb force on this particle directed roughly from a towards b, or from b towards a? (b) What is the sign of the coulomb electric work done on this particle as it moves from b to a? *(Answer: 28) (Suggestion: [s-1])*

Understanding $RI = V + \mathcal{E}$ (Cap. 1d)

| F-2 | *Example:* As current flows from b to a through the battery shown in Fig. F-2, the *coulomb* work per unit charge is $V = -1.4$ volt, while the *non-coulomb* work (supplied by the chemical reactions in the battery) is $\mathcal{E} = 1.5$ volt. What is the total work per unit charge done on particles moving from b to a through this battery? If the resistance of the battery is 0.1 ohm, what is the current I? *(Answer: 25)*
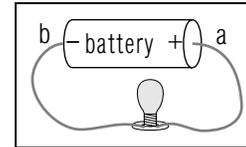
Fig. F-2.

| F-3 | *Properties:* Answer these questions for potential drop, emf, and resistance: (a) What algebraic symbol most often represents this quantity? (b) Is this quantity a number or a vector? If it is a number, what signs can it have $(+, -, 0)$? (c) What is the SI unit of this quantity? *(Answer: 31)*

| F-4 | *Dependence of work per unit charge on resistance:* A thin wire A has a resistance equal to four times the resistance of a thick wire B. (a) Suppose the same current I flows through both wires. To compare the works w_A and w_B per unit charge required to produce these currents; state whether the work per unit charge w_A for wire A is equal to 4, or 1/4 times the work per unit charge w_B for wire w_B. (b) For a wire with large resistance, is the work per unit charge w larger or smaller than the work per unit charge required to produce the *same* current through a wire with smaller resistance? *(Answer: 34)*

Knowing About Energy Transfer

| F-5 | Consider an electron in a wire which is connected to the terminals of a battery. As the electron moves along the wire, electric work is continually done on it, and so its electric potential energy decreases. Yet in a steady state, the average speed v of the electron remains constant, and so its average kinetic energy, $(1/2)mv^2$, does *not* increase. (a) What kind of energy *does* increase as the electron's potential energy decreases? (b) Use your answer to explain why a lit light bulb is hot. *(Answer: 29)*

SECT.

G SUMMARY

DEFINITIONS

electric current; Def. (A-1)

ampere; Eq. (A-2)

steady state; Def. (C-1)

two-terminal system; Def. (D-1)

series connection; Sec. D, Fig. D-3a

parallel connection; Sec. D, Fig. D-3b

emf; Def. (F-2)

electric resistance; Def. (F-5)

ohm; Eq. (F-6)

IMPORTANT RESULTS

Definition of electric current: Def. (A-1)

$I = dQ/dt$

Relation between current and charged particle motion: Eq. (B-2)

$I = (N/l)qv$

Relation between currents in a steady state: Eq. (C-2), Eq. (E-1)

For any region, $I_{in} = I_{out}$

Relation between potential drops: Rule (E-2)

V_{ab} is the sum of the potential drops along *any* path from a to b.

Relation between I and V for a two-terminal system: Eq. (F-4)

$RI = V + \mathcal{E}$ (if I is not too large)

USEFUL KNOWLEDGE

Effective currents and potential drops (Sec. C)

circuit diagrams (Sec. D)

Energy transfer in circuits (Sec .F)

NEW CAPABILITIES

(1) Understand these relations:

(a) The definition $I = dQ/dt$ of current (Sec. A);

(b) $I_{in} = I_{out}$ for currents in a steady state (Sects. C and E, [p-3]);

(c) V_{ab} = sum of potential drops along any path from a to b (Sec. E);

(d) $RI = V + \mathcal{E} = w$, the total work per unit charge (Sec. F).

(2) Relate a current's magnitude and sense (or sign relative to a chosen sense):

(a) To a description of the average motion of charged particles (across a boundary or along a wire.) (Sects. A and C, [p-1]).

(b) To the number of moving charged particles per unit length of wire, the average velocity of these particles, and the sign of their charges. (Sec. B, [p-2]).

(3) Use $I_{in} = I_{out}$ and V_{ab} = sum of potential drops along any path to relate the currents and potential drops in a circuit. (Sects. C, D, and E, [p-4], [p-8]).

(4) Use interchangeably the following descriptions of a circuit:

(a) Picture or drawing of the circuit;

(b) Various equivalent circuit diagrams;

(c) Verbal description of the circuit in terms of parallel and series connections. (Sec. D, [p-5], [p-6], [p-7]).

Tutorial aids:

Sec. C: Visualizing currents

Sec. D: Relating circuits described in various ways

Sec. E: Relating currents and potentials in a circuit

Sec. H: Additional problems

Relating Descriptions of Current (Cap. 1,2,3)

G-1 The belt carrying charge to a large Van-de-Graaff generator has a width of 50 cm and a charge density of 6.6×10^{-6} C/m². Thus the charge Q in a 1 meter length of belt is 3.3×10^{-6} C. If the belt moves at a speed of 30 m/s, what is the magnitude of the current due to the charge carried by the belt? What is the charge delivered by the belt to

the Van-de-Graaff sphere during a time of 0.10 minute? *(Answer: 37)* *(Suggestion: [s-4])*

G-2 A current of 0.1 ampere flows into the bulb of a flashlight. How much charge and how many electrons (each with charge -1.6×10^{-19} C) flow into (and out of) this bulb during 5 minutes? *(Answer: 40)*

Relating Currents and Potentials In a Circuit (Cap. 4)

G-3 Two two-terminal systems A and B can be connected in series or in parallel. For which kind of connection is the potential drop across the entire system simply equal to the potential drop across A (or B)? For which kind of connection is the current into (or out of) the entire system simply equal to the current through A (or B)? *(Answer: 35)*

G-4 A toaster and a coffee-maker are both plugged into a wall socket so that the two prongs on the plug of each appliance are connected to the "line" wires leading ultimately to an electric "generator." (a) Are the toaster and coffee-maker connected in series or in parallel? How can you tell? (b) Make a circuit diagram showing the connections of these two appliances to the "line" wires. *(Answer: 32)*

G-5 There is ordinarily an effective potential difference of 110 volt between the two "line" wires connected to a household wall socket. In the situation described in problem G-4, suppose that effective currents of 8.3 ampere and 6.4 ampere flow through the toaster and through the coffee-maker. (a) What is the effective potential difference between the terminals of the toaster and of the coffee-maker? (b) What is the effective current through each of the "line" wires connected to the wall socket? *(Answer: 38)*

SECT.

H PROBLEMS

H-1 *Measuring Avogadro's number:* Avogadro's number N is defined as the number of atoms (or molecules) in one mole (or one gram-atomic-mass) of a pure substance. For example, there are N atoms of silver in 108 grams of pure silver metal. The number N is most accurately measured in this way: The terminals of a battery are connected to two metal plates immersed in a solution containing ions, for example Ag^+ (silver) and Cl^- (chloride). (See Fig. H-1.) Positively charged Ag^+ ions move to the negatively charged plate P, where they each acquire an electron. and so become atoms of silver which are deposited on the plate P. For every electron flowing through the surface of P, exactly one atom of silver is deposited. If a current of 1.0 ampere flows through the solution for 8.0 minute, 0.54 gram of silver is deposited. (a) How much time is required to deposit 54 gram and to deposit 108 gram (or one mole) of silver? (b) During the time required to deposit one mole of silver, how many coulombs of charge and how many electrons pass through the surface of the plate P? (c) Use your preceding answers to find a value for N. *(Answer: 39)*

H-2 *A clever circuit:* Light fixtures for long halls or stairways are commonly controlled by two switches, one at each end of the hall. The switches are connected as shown in Fig. H-2 so that when the light is on, either switch can turn it off; and when the light is off, either switch can turn it on. (a) Describe the positions (up or down) of the switches A and B which result in current flowing through the light (i.e., in the light being "on.") (b) When the light is "on," what changes in the position of either switch result in turning the light "off"? *(Answer: 36)*

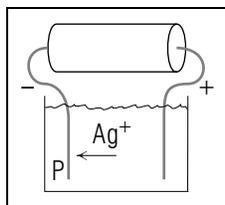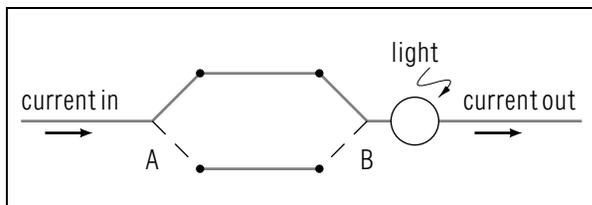*Note: Tutorial section H includes additional problems.*

Fig. H-1.   Fig. H-2.

TUTORIAL FOR D

RELATING CIRCUITS DESCRIBED BY DIAGRAMS, WORDS, AND PICTURES (Cap. 4)

d-1 *PURPOSE:* Circuits are described in various ways, There are photographs (or pictorial drawings) of the real circuit, showing various two-terminal systems (e.g., batteries, lamps) connected together with real wires. There are circuit diagrams which use labeled boxes (or other symbols) to indicate various two-terminal systems, and which use lines to connect terminals which have the same potential and between which current can flow. Finally circuits can he described by using words such as "series" and "parallel." To make matters even more complex, the same "real" circuit can be described by circuit diagrams which look very different. (See for example Fig. D-2 in the text.)

The following frames should help you to use and to translate between all of these different representations of circuits. In particular, you should learn how to tell when two different descriptions are "equivalent," i.e., when they describe the same real circuit.

d-2 *HOW TO RECOGNIZE EQUIVALENT CIRCUIT DIAGRAMS:* Two circuits are equivalent if all connections are the same. To decide whether two circuits are equivalent, one just has to consider in turn each terminal and check that in each circuit this terminal is connected to the same other terminals. For example, let us compare the various connections in circuit diagrams (1) and (2):
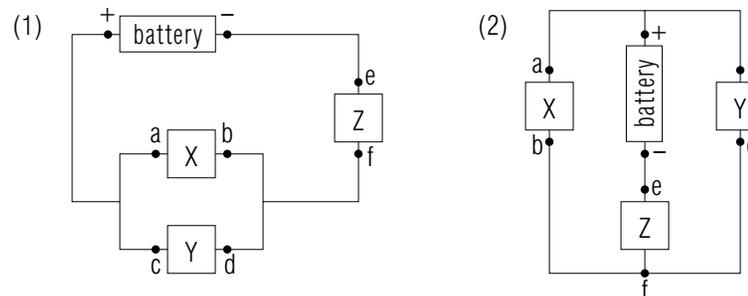
In the following chart, the first column lists the eight terminals of the four two-terminal systems appearing in each of the preceding circuit diagrams. The second and third columns list, for each terminal, the connections

shown in the circuit diagrams (1) and (2). For example, in both diagrams, the positive terminal of the battery is connected to the terminals a and c of systems X and Y.

Complete the following chart by writing appropriate labels for terminals.
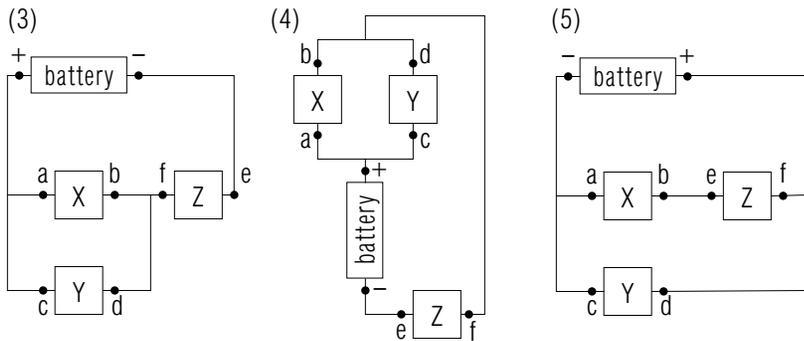
▶

Terminal		Connections	
		(1)	(2)
Battery +		to a and c	to a and c
	−	to e	to e
X	a	to + and c	to + and c
	b	to d and f	to d and f
Y	c		to + and a
	d		to b and f
Z	e	to −	
	f	to b and d	

Are the preceding two circuit diagrams equivalent?

▶ yes, no

(Answer: 58)

d-3 *PRACTICE IN RECOGNIZING EQUIVALENT CIRCUITS:*

(3)   (4)   (5)

Complete the following chart by writing, for each of the terminals listed in the first column, the terminals to which it is connected in each of the preceding circuit diagrams (3), (4), and (5).

▶

Terminal		Connections		
		(3)	(4)	(5)
battery +				
	−			
X	a			
	b			
Y	c			
	d			
Z	e			
	f			

Which preceding circuit diagram (or diagrams) is equivalent to the diagrams (1) and (2) shown in the preceding frame?

▶ (3), (4), (5)

(Answer: 53)

d-4 *RELATING CIRCUIT DIAGRAMS TO PICTORIAL DRAWINGS:* The following drawing shows a real battery, some light bulbs, and a mysterious black box all connected together with wires. For the accuracy desired here, terminals connected by wires have the same potential.
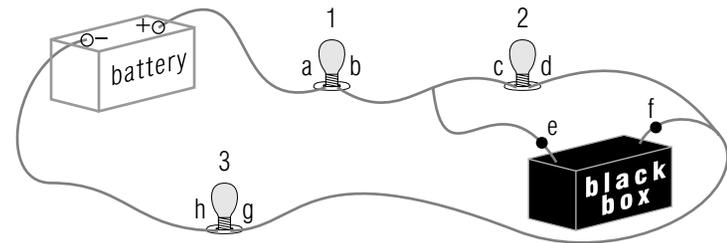
Which of the following circuit diagrams correctly describes the connections in the drawing of the real circuit?

(a)

(b)
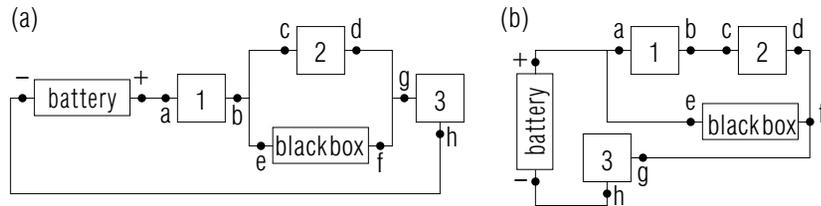
► (a), (b)

(Answer: 56) (Suggestion: [s-9].)

d-5   *DESCRIPTIONS OF SERIES AND PARALLEL CONNEC-TIONS:* Two two-terminal systems are connected in *series* if successive terminals are connected together so that charged particles move first through one system and then through the other along a single path. (See Fig. D-3 in the text.)

Two two-terminal systems are connected in *parallel* if pairs of corresponding terminals are connected so that some charged particles flow through one system and the rest flow through the other. (See Fig. D-3.)

The following drawings show two light bulbs connected in various ways (any one drawing might show series, parallel, or neither).

(a)

(b)

(c)

(d)

(e)
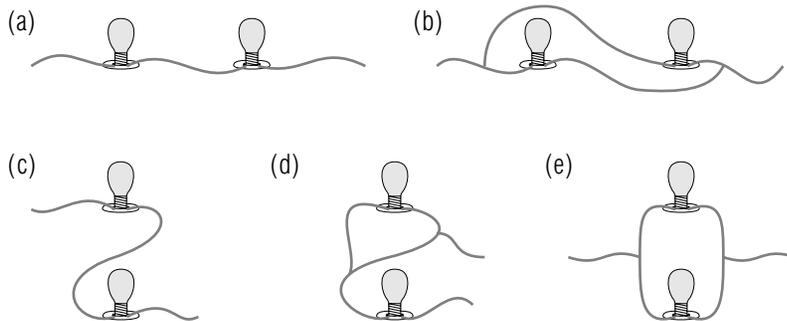
Which of the preceding diagrams show the bulbs connected in series and which show the bulbs connected in parallel?

► series: (a), (b), (c), (d), (e)?

parallel: (a), (b), (c), (d), (e)?

(Answer: 62) (Suggestion: [s-12].)

d-6   *SUMMARY:* Two circuits (or diagrams describing circuits) are *equivalent* if all connections between terminals are the same. Thus recognizing equivalent circuits requires consideration of each terminal in turn, and checking that it is connected in the same way in both circuits. The spatial arrangement of these systems can be ignored, because it does not affect whether the two circuits are equivalent.

The terms series and parallel are used to describe special connections between systems (not how these systems are arranged in space).

*Now: Go to text problem D-3.*

TUTORIAL FOR E

RELATING CURRENTS AND POTENTIALS IN A CIRCUIT (Cap. 3)

e-1   *A SYSTEMATIC STRATEGY:* Relating all the various currents and potentials in a circuit can be a complex process. Therefore the following systematic strategy can be helpful.

(1) *To describe the problem*, draw a circuit diagram (if none is provided) and list symbols (and known values) for both known and desired quantities. The circuit diagram should include algebraic symbols (e.g., I, I_1) and sense arrows describing all currents in the system.

(2) *To relate currents*, apply $I_{in} = I_{out}$ to junctions, places where various different currents flow into and out of a region.

(3) *To relate potentials*, first choose and label points (e.g., a and b) at which the potentials are of interest (or between which the potential drops are of interest). Then relate potential drops by applying $V_{ab} =$ sum of potential drops along any path from a to b.

e-2   *A SAMPLE PROBLEM:* Let us consider one sample problem, and see how the preceding strategy can be applied to solve it.

Problem: The following drawing shows a "battery pack." Three 1.5 volt batteries are held, side-by-side, in a box with metal ends which are not connected to each other, but which extend across the corresponding terminals of all the batteries. The metal ends of the battery pack are then connected to a light bulb and to a small motor, which are connected in series. A current of 1.0 ampere flows through the motor and the potential drop across it is 0.7 volt.

sense of the current
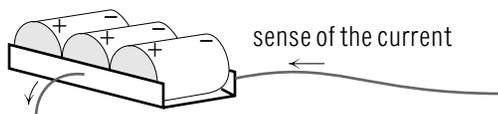
Let us apply our strategy to find the potential difference between the terminals of the light bulb, the current through this bulb, and the current through each battery.

(1) *Describe*

This problem will be easier to solve if we have a convenient circuit diagram and an organized summary of known and desired information, including symbols for all quantities. Let us label the batteries by 1, 2, and 3, and use the symbols I_1, I_2, I_3, and I to represent the currents through these batteries and through the motor.
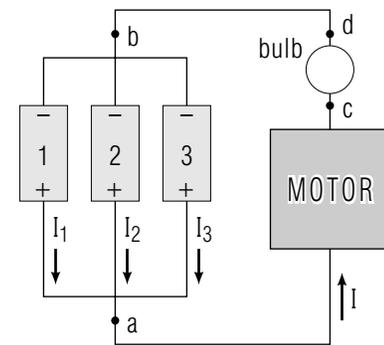
Known:

$I = 1$ ampere
potential drop across each battery is 1.5 volt
potential drop across motor is 0.7 volt

Desired:

I_1, I_2, I_3
current through light bulb
potential drop across light bulb

(2) *Relate currents*

Because the current I through the motor flows out of the motor and directly into the light bulb

$$\text{current through light bulb} = I = 1 \, \text{ampere}.$$

Because the three batteries are identical and are connected in exactly the same way, $I_1 = I_2 = I_3$.

To find the desired value for I_1, I_2, I_3, we need only apply $I_{in} = I_{out}$ to the junction near the point a.

$$I_{in} = I_{out}$$

$$I_1 + I_2 + I_3 = I$$

Therefore

$$I_1 = I_2 = I_3 = \frac{1}{3}I = \frac{1}{3}\,\text{ampere}$$

(3) *Relating potentials*

We can use the points a, b and c indicated on the diagram to describe the potential drops of interest in this problem. Remember that all points connected by lines to the point a (or b) have the same potential. Thus the potential drop across one of the batteries (say battery 2) is $V_{ab} = 1.5\,\text{volt}$. The potential drop across the motor is $V_{ac} = 0.7\,\text{volt}$. The desired potential drop V_{cb} across the light bulb can then be found in this way:

$$V_{ab} = \text{sum of potential drops along any path from } a \text{ to } b$$

$$1.5\,\text{volt} = V_{ac} + V_{cb}$$

$$1.5\,\text{volt} = 0.7\,\text{volt} + V_{cb}$$

$$V_{cb} = 0.8\,\text{volts}$$

Now: Return to text section E, and apply the strategy illustrated in this frame to solve problem E-1, E-2, and E-3.

TUTORIAL FOR H

ADDITIONAL PROBLEMS

The following problems concern capacitors considered as two-terminal systems which can be connected in series and in parallel.

h-1 *CAPACITORS AS TWO-TERMINAL SYSTEMS:* When charge is transferred from one plate of a capacitor to the other, current flows *into* one plate (causing it to become more positive) and flows *out of* the other plate (causing it to become more negative). Usually current flows into and out of the capacitor through wires attached to the plates. Thus a capacitor is a two-terminal system (although, when the capacitor is fully charged, no current flows through these terminals). Thus in studying these systems we shall focus attention on the potential differences between the terminals and on the charge transferred between the plates (rather than on the current flowing through the system).

The following symbol is used in circuit diagrams to represent a capacitor with terminals a and b.
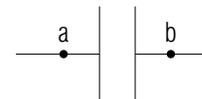

In the space below, draw a circuit diagram showing two capacitors connected in parallel.

▶

(Answer: 67)

h-2 *CAPACITORS CONNECTED IN PARALLEL:* When two capacitors are connected in parallel, as shown in the following diagram, the

result is a new capacitor. The reason is that the two plates connected to the terminal a are really just one connected conductor (as are the two plates connected to b). Thus the combined system is just two conductors arranged such that charge can be transferred out of one conductor and into the other.
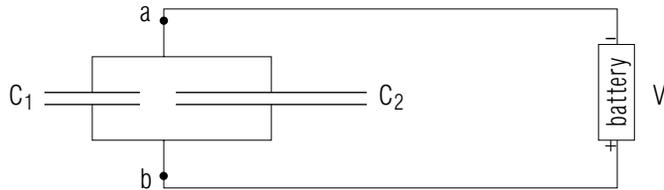
We can relate the capacitance C of the combined system to the capacitances C_1 and C_2 of the individual capacitors which make it up in this way:

Suppose a battery producing a potential difference V between its terminals is connected to the terminals a and b in the preceding diagram so that a charge Q is transferred from a to b. Then the capacitance C of the combined system is just Q/V. (a) Write an equation relating the potential difference V between the terminals a and b of the combined system, and the potential differences V_1 and V_2 between the terminals of the individual capacitors. (b) Write an equation relating the total charge Q transferred between the terminals a and b and the charges Q_1 and Q_2 transferred between the plates of the individual capacitors. (c) Use the results of parts (a) and (b) to write an equation relating the capacitances Q/V, Q_1/V_1 and Q_2/V_2 of the combined system and of the individual capacitors. Re-express this equation in terms of the symbols C, C_1 and C_2. *(Answer: 64)*

h-3 *CAPACITORS IN SERIES:* The following diagram shows two capacitors connected in series and then connected to a battery so that a (positive) charge Q is transferred out of the top plate of capacitor 1 and into the bottom plate of capacitor 2. The terminals a and b have a potential difference of V.
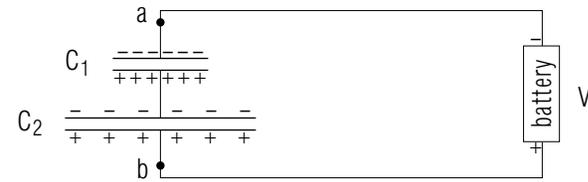
Notice that the remaining plates of each capacitor make up the central "I-shaped" piece which is insulated from other objects. Thus the total charge of these connected conductors must remain zero. The top conductor (the bottom plate of 1) then acquires a charge of $+Q$, while the bottom conductor (the top plate of 2) acquires a charge of $-Q$.

(a) Write an equation relating the total charge Q transferred between the terminals a and b of the combined system and the charges Q_1 and Q_2 transferred between the terminals of the individual capacitors. (b) Write an equation relating the potential difference V between the terminals a and b of the combined system and the potential differences V_1 and V_2 between the terminals of the individual capacitors. (c) Use the results of parts (a) and (b) to relate the ratios V/Q, V_1/Q_1, and V_2/Q_2. Re-express this equation as a relation between $1/C$, $1/C_1$, and $1/C_2$. *(Answer: 61)*

h-4 *DANGEROUS VOLTAGE?:* A plastic ball of radius 1 cm is rubbed with plastic film and so acquires a uniformly distributed charge of 1×10^{-8} C. What is the potential produced by this charge at the surface of the ball? Why is it not dangerous to touch this "high voltage" ball? [There is a similarly large potential produced at the surface of a comb run through dry hair.] *(Answer: 68)*

PRACTICE PROBLEMS

p-1 *RELATING CURRENT SIGN AND SENSE TO MOTION (CAP. 2A):* The following diagram shows charged particles in an ion "beam" moving through the evacuated tube in an ion accelerator.
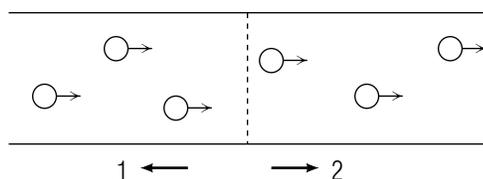
(a) If the particles are positively charged, which of the arrows (1 or 2) indicates the sense of the "beam current" through the surface indicated by a dotted line? If arrow 2 indicates a chosen sense, what is the sign of this current? (b) Suppose the particles are negatively charged. Which arrow then indicates the sense of the current? What is the sign of this current relative to a chosen sense indicated by arrow 2? *(Answer: 52) (Suggestion: Review text problems A-4, A-5, and A-6.)*

p-2 *RELATING CURRENT MAGNITUDE TO PARTICLE MOTION (CAP. 2B):* Increasing the potential difference between the ends of a wire (e.g., by connecting it to a battery with larger emf) increases the magnitude of the electric force on each mobile electron, and so increases the average velocity of these electrons. Does increasing the potential difference between the ends of a wire result in a current which is larger or smaller in magnitude? *(Answer: 55) (Suggestion: Review text problem B-1.)*

p-3 *UNDERSTANDING $I_{IN} = I_{OUT}$: ORGANIZING RELATIONS (CAP. 1B):* The following diagram shows the surface of a lead plate in an automobile storage battery. Positively charged lead ions leave the surface of the metal (and combine with sulfur ions to form lead sulfide).
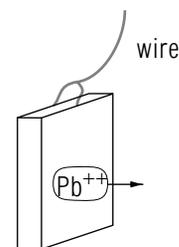

(a) Suppose that the lead plate is in a steady state, and that the lead ions leaving the surface during 0.5 second have a total charge of 2.4×10^{-4} coulomb. During this time what is the charge moving into the lead plate (from a wire attached to the terminal of the battery)? (b) In fact, the mobile charged particles within the lead plate (and the wire attached to it), are negatively charged electrons. Do electrons move from the plate into the wire or from the wire into the plate? *(Answer: 59) (Suggestion: Review text problems C-1 and C-2.)*

p-4 *RELATING CURRENTS (CAP. 3):* The following diagram shows the magnitudes and senses of some of the currents flowing in various wires connecting a battery and four light bulbs. What is the magnitude and sense of the current through light bulbs 2 and 4? *(Answer: 54) (Suggestion: Review tutorial section C plus text problems C-6 and C-7.)*
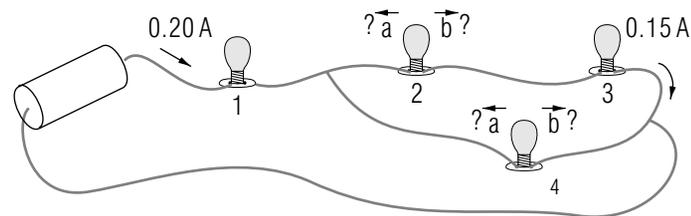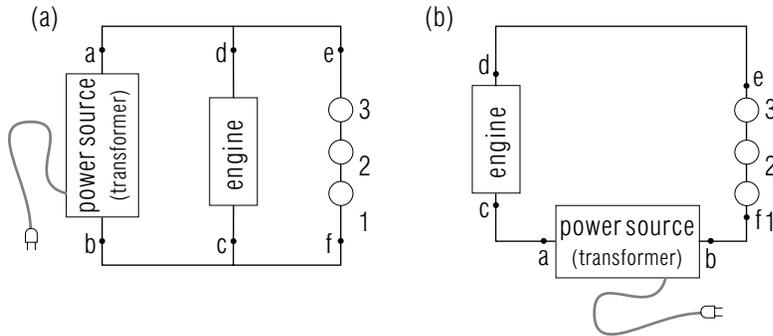

p-5 *RELATING DESCRIPTIONS OF CURRENTS (CAP. 4):* The following drawing shows a two-terminal system which is part of the wiring for an electric toy train. Current enters and leaves the system through the two wires leading to the wall socket. The transformer has two additional terminals on its top through which current can flow in or out. These terminals are connected by wires to the train so that current flows along the track, through the toy engine, and back along the track

to the transformer. Two other wires pass under the track (but are not connected to it) and lead to the row of three lights.
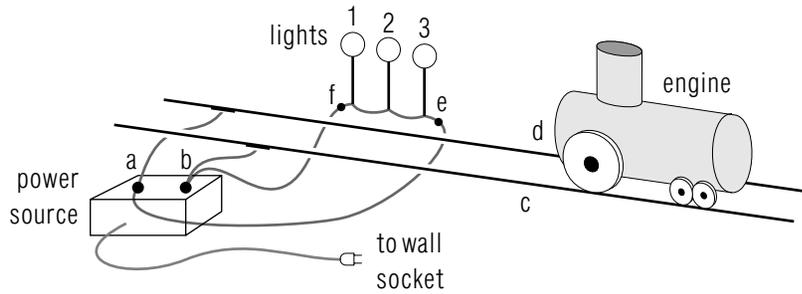

Which of the following circuit diagrams correctly describes the wiring shown in the preceding drawing? *(Answer: 66) (Suggestion: Review text problem D-5 and tutorial frames [d-3] and [d-4].)*

p-6 *RELATING DESCRIPTIONS OF CURRENTS (CAP .4):* To answer the following questions, refer to the circuit diagrams in practice problem

p-5 *,:* and decide whether the connection described is shown in diagram (a), in diagram (b), in *both* of these diagrams, or in *neither* of these diagrams. (1) In which diagram(s) is the engine connected in parallel with the system consisting of the three lights? (2) In which diagram is the engine connected in series with light 3? *(Answer: 63) (Suggestion: Review text problems D-7 and D-8, and tutorial frame [d-5].)*

p-7 *RELATING DESCRIPTIONS OF CIRCUITS (CAP. 4):* The following picture shows two batteries connected to an "ammeter," an instrument which measures the current flowing between its two terminals. (a) Are the two batteries connected in parallel, in series, or neither in parallel nor in series? (b) Use labeled rectangles to draw a circuit diagram for this circuit. *(Answer: 60) (Suggestion: Review text problems D-6 and D-7.)*
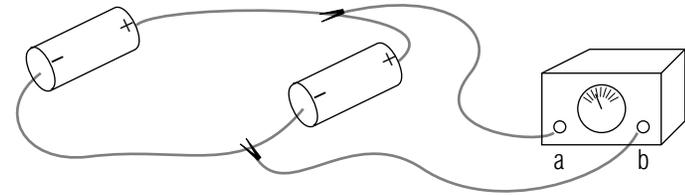
p-8 *RELATING CURRENTS AND POTENTIALS IN A CIRCUIT (CAP. 3):* Two batteries are connected together in parallel so as to form a two-terminal system with one terminal connected to the positive terminals of both batteries and one terminal connected to the negative terminals of both batteries. These two terminals are connected to the terminals of a battery-operated hand-warmer. If the potential drop across each battery is 1.5 volt, and the current through each battery has a magnitude of 0.2 ampere, what is the magnitude of the current through the hand-warmer? What is the magnitude of the potential drop across it? *(Answer: 65) (Suggestion: Review text problems E-1, E-2, and E-3, and tutorial frames [e-1] and [e-2].)*

SUGGESTIONS

s-1 *(Text problem F-1):* Notice that a positively charged particle within the battery is acted on by a repulsive electric force due to the positively charged terminal, and by an attractive force due to the negatively charged terminal.

s-2 *(Tutorial frame [c-2]):* Imagine a stream of particles flowing from A to B (or from B to A). If no particles leave or join this stream between A and B, then the currents I_A and I_B must be equal. If particles do leave or join this stream, then ordinarily I_A and I_B are not equal.

s-3 *(Text problem D-3):* In a circuit diagram, lines are used to connect points which must have the same potential because they are connected (e. g ., by a short wire).

s-4 *Text problem G-1):* Consider an imaginary surface through which the belt moves. Then the current carried by the belt is dQ/dt, where dQ is the amount of charge moving through the surface during the time dt.

If there is 3.3×10^{-6} C of charge on each meter of belt, and if the belt moves at 30 m/s, then in one second, $(30)\,(3.3 \times 10^{-6})$ (C) of charge move through this surface.

s-5 *(Text problem C-3):* Remember that the sense of a current is always towards the region in which charge increases, either due to the arrival of positive particles or to the departure of negatively charged particles. Thus the moving negatively charged particles shown in the following diagram produce a current with the sense indicated.
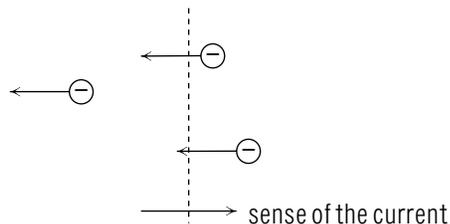

s-6 *(Text problem C-8):* Because of the open switch, the current out of the coffee-maker is zero. But then according to $I_{\text{in}} = I_{\text{out}}$, the current into the coffee-maker is also zero. (After all, if this current were not zero, charge would pile up in the coffee-maker!)

A single, unbroken wire leads from the wall outlet, through the toaster, and back to the wall outlet. Thus current can flow along this continuous path (even though the switch S is open).

s-7 *(Text problem C-7):* For simplicity, suppose that the current is due to moving positively charged particles. Then particles producing a current of 12.2 A flow out of the coffee maker. At the first junction, these particles are joined by particles flowing out of the toaster, resulting in a current of 12.2 A + 8.3 A flowing to the left from this junction. At the second junction, this current of 20.5 A is joined by particles flowing out of the refrigerator to make up the current I flowing out of the kitchen.

s-8 *(Text problem C-2):* (a) The following diagram shows the capacitor plate with a positive current flowing into the region consisting of this plate.
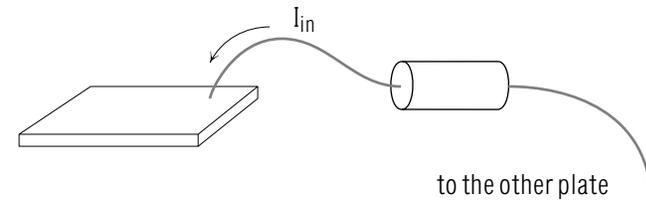
As the current flows into the region of the plate, the charge of this region increases. Thus this region is not in a steady state (because its charge is not constant). Hence the current in certainly does *not* equal the current out.

(b) When the plate is fully charged, no further charged particles move into it. The charge of the region then remains constant, and $I_{\text{in}} = I_{\text{out}} = 0$ in this steady-state situation.

s-9 *(Tutorial frame [d-4]):* To decide which of these diagrams is equivalent to the circuit, follow the procedure illustrated in tutorial frames [d-2] and [d-3]. Systematically consider each terminal, and check whether the connections are the same in the diagram as in the circuit. For example,

in the real circuit, terminal b of light bulb 1 is connected to terminals c and e.

With practice you can probably make these comparisons visually (without filling out a complete chart). However, it is always essential to compare all connections between terminals.

s-10 *(Tutorial frame [c-3]):* To answer questions like this, imagine fluid currents flowing in an analogous set of connected pipes. For each of the two possible senses of the unknown current, ask, "Would a current having this sense result in currents flowing into a region from which no currents flow out?" If so, the situation is impossible, and the unknown current can *not* have this sense. [Similarly, the unknown current cannot have a sense which results in currents flowing *out* of a region for which no currents flow *in*.

For example, as the following diagram indicates, the unknown current diagram (b) cannot have sense 1.

region with current out
but no current in

1
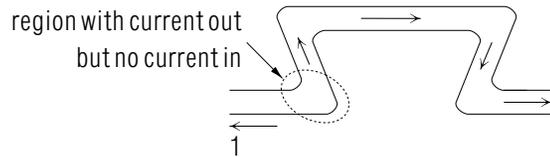
s-11 *(Text problem A-4):* (a) Remember that the sense of the current is towards the region in which positive charge increases (due to motion across the surface considered). This increase can be due either to the arrival of positively charged particles, or to the departure of negatively charged particles.

(b) Relative to a chosen sense, the sign of a current is positive if its sense is the same as the chosen sense. If these senses are opposite, then the current is negative.

s-12 *(Tutorial frame [d-5]):* In each drawing, the two terminals of the system are indicated by wires which stick out. Imagine that the current flows in through one of these terminals and out through the other, and then think about the path or paths along which charged particles move. If they move along one path successively through both light bulbs, then the bulbs are connected in series. If the stream of charged particles sep-

arates, part going through one light bulb and part through the other, then the bulbs are connected in parallel. If neither of the preceding statements describes the paths followed by charged particles, then the bulbs are connected neither in parallel nor in series.

s-13 *(Text problem E-2):* The following circuit diagram describes the two light bulbs connected in series.

a   1   c   2   b

2.0 A ⟶
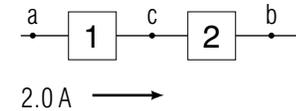
Because the current flowing into the system is 2.0 ampere, the current through each bulb is also 2.0 ampere.

The potential drop V_{ab} across the entire system is equal to the sum of the drops V_{ac} and V_{cb} across the individual light bulbs.

$$V_{ab} = V_{ac} + V_{cb}$$

$$100 \, \text{volt} = 70 \, \text{volt} + V_{cb}$$

Thus the desired potential drop is $V_{cb} = 30 \, \text{volt}$.

s-14 *(Text problem D-6):* (a) If the current flows in through bulb 1, then it simply continues through 2 and then 3, finally leaving the system through the long wire. Thus the following diagram simply describes the connections of the bulbs in Fig. D-8a.

a   1   b   c   2   d   e   3   f
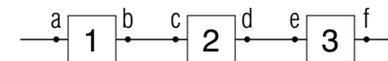
(b) Corresponding terminals from each bulb are connected together. All of the light bulbs are connected to the wire through which current flows in and out. Thus either of the following diagrams correctly describes the

connections in Fig. D-8b.
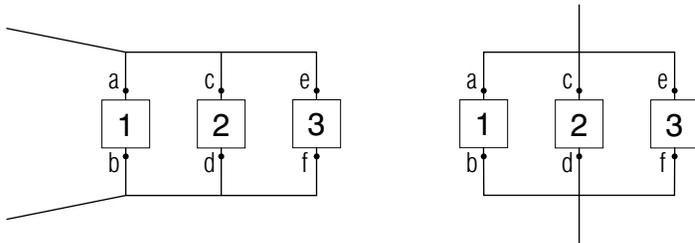
ANSWERS TO PROBLEMS

1. a. $I_1 = 0.6\,\text{A}$, $I_2 = -0.2\,\text{A}$, $I_3 = -0.4\,\text{A}$ (Note signs)

   b. $dQ = 0$

2. $1.8 \times 10^{-4}\,\text{m/s} = 0.018\,\text{cm/s}$

3. a. B and E

   b. A and D

4. a. Q, number, $+$, $-$, 0, coulomb (C); I, number, $+$, $-$, 0, ampere (A)

   b. 1 ampere

5. along, opposite

6. a. ions, 2; electrons, 2

   b. negative, negative

7. a. $I = dQ/dt$

   b. 3.2×10^{-7} coulomb, 8.0×10^{-5} ampere

8. a. 2

   b. opposite

9. $I_{\text{in}} = I_{\text{out}} = 71$ ampere

10. a. zinc, 1; carbon, 1

    b. positive, positive

    c. negative, negative

11. a. Plate is not in a steady state because its charge increases.

    b. Plate is now in a steady state.

12. a. decreases

    b. larger

13. 27.1 ampere

14. b and c

15. a. 1

    b. 1

    c. 2

d. either

e. 2

16. a, b

17. +: a, b ; −: d

18. 3 ampere, with a sense out of the indicated region.

19. a. decreases

b. remains the same

c. loses energy to the atoms of the wire (which increase in random internal energy)

20. Diagrams equivalent to:

(a)

(b)

fluorescent

incandescent
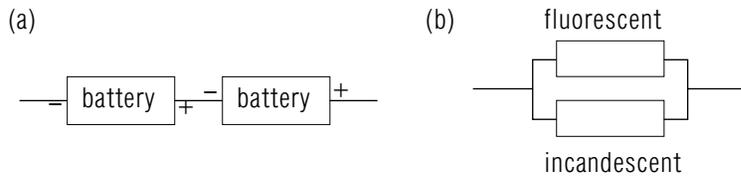
21. 30 volt, 2.0 ampere. If wrong, see [s-13].

22. a, yes

23. a, b or c

24. a. 12 volt

b. 0.9 A through each

25. 0.1 volt, 1 ampere

26. a. Diagram should be simple and show that all charged particles pass through each light bulb sequentially. For example, diagram (a) is correct.

b. Diagram should be simple and show that all the bulbs are connected at one terminal to the wire through which current flows in, and at the other terminal to the wire through which current flows out. For example, diagram (b) is correct.

(a)

(b)
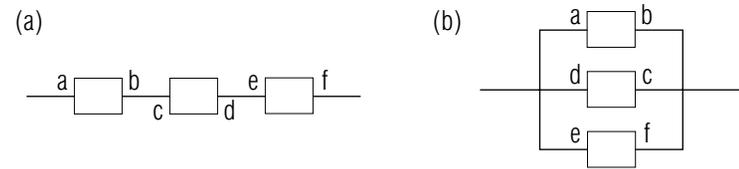
If wrong, see [s-14].

27. a. diagram equivalent to:
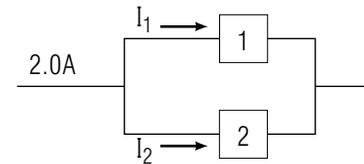
b. 0.6 ampere

c. 100 volt

28. a towards b, negative

29. a. random internal energy of electron and wire

b. random internal energy *is* heat

30. series: battery and 2. parallel: 1 and 3. neither: 2 and 3.

31.

V or V_{ab}	number (+, −, 0)	volt (V)
$\mathcal{E}$	number (+, −, 0)	volt (V)
R, r	number (+, 0 [super-conductor])	ohm (Ω)

32. a. parallel. If in series, unplugging one would stop current flowing and through the other.

b. simple diagram equivalent to:

line

wires

toaster coffee maker
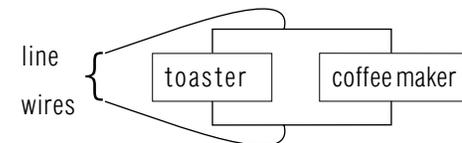
33. a. series

b. parallel

c. parallel

d. series

34. a. 4

b. larger

35. parallel, series

36. a. Light is on if both switches are up or if both are down.

b. Changing position of either switch turns light off.

37. 9.9×10^{-5} A or 1.0×10^{-5} A, $(5.9$ or $6.0) \times 10^{-4}$ coulomb

38. a. 110 volt, 110 volt

b. 14.7 A

39. a. 800 minute, 1600 minute $= 9.6 \times 10^4$ sec

b. 9.6×10^4 coulomb, 6.0×10^{23} electrons

c. 6.0×10^{23}

40. 30 coulomb, 1.9×10^{20} electrons

51. 1,2, either

52. a. 2, positive

b. 1, negative

53.

	(3) & (4)	(5)
$+$	a and c	d and f
$-$	e	a and c
a	$+$ and c	$-$ and c
b	d and f	e
c	$+$ and a	$-$ and a
d	b and f	$+$ and f
e	$-$	b
f	b and d	$+$ and d

3 and 4 equivalent to 1 and 2

54. (2) 0.15 A, sense b. (4) 0.05 A, sense b.

55. larger

56. a

57. yes, yes, no

58. In diagram (1), c is connected to $+$ and a, d is connected to b and f. In diagram (2), e is connected to $-$, f is connected to b and d. Yes.

59. a. 2.4×10^{-4} coulomb

b. from plate to wire

60. a. parallel

b. diagram simple and equivalent to:
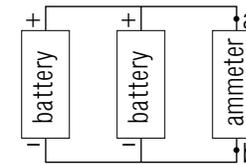
61. a. $Q = Q_1 + Q_2$

b. $V = V_1 + V_2$

c. $V/Q = V_1/Q_1 + V_2/Q_2$, $1/C = 1/C_1 + 1/C_2$

62. series: a, c ; parallel: b, e

63. (1) a, (2) b

64. a. $V = V_1 = V_2$

b. $Q = Q_1 + Q_2$

c. $Q/V = Q_1/V_1 + Q_2/V_2$, $C = C_1 + C_2$

65. 0.4 ampere, 1.5 volt

66. (a)

67. Each terminal of one capacitor should be connected to one terminal of the other capacitor, as in this diagram:
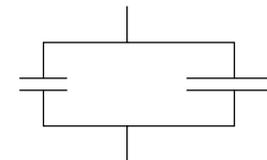
68. 9×10^3 volt. Because the total charge is so small, any circuit flowing due to this potential stops almost immediately.

MODEL EXAM

1. **Relating charge and current.** If a steady current of 1 ampere flows through a surface for one minute (60 second), how much charge passes through the surface?

2. **Direction of motion of electrons in a wire.** In a wire, the current is carried by moving electrons (particles with negative charge). The diagram below shows a section of such a wire.
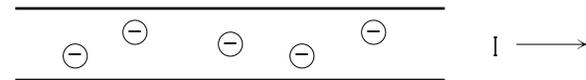

In which sense, right or left, must the electrons move so that the current will have the sense shown in the diagram?

3. **Relating currents and potential drops in a circuit.** The following diagram shows a circuit, and indicates values for some of the currents and potential drops describing this circuit.
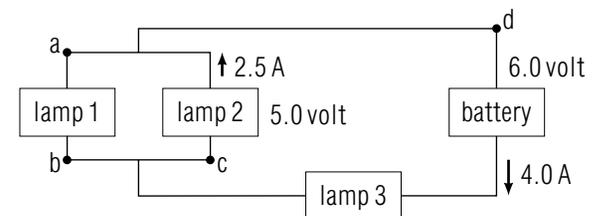
a. What are the magnitude and the sense (a to b or b to a) of the current through the lamp 1?

b. What is the magnitude of the potential drop across lamp 1?

c. Which of the points b, c, and d indicated on the diagram must have the same potential as the point a?

4. **Describing a circuit in words.** The following diagram shows a two-terminal system consisting of three connected light bulbs (each bulb is indicated by a circle).
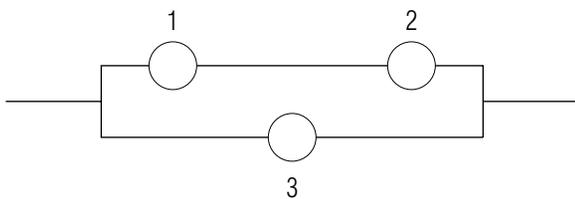
Are light bulbs 2 and 3 connected in series, connected in parallel, or connected neither in parallel nor in series?

Brief Answers:

1. 60 coulomb or 6×10^1 coulomb

2. left

3. a. Magnitude: $I = 1.5$ ampere, Sense: b to a

   b. 5.0 volt

   c. d

4. neither