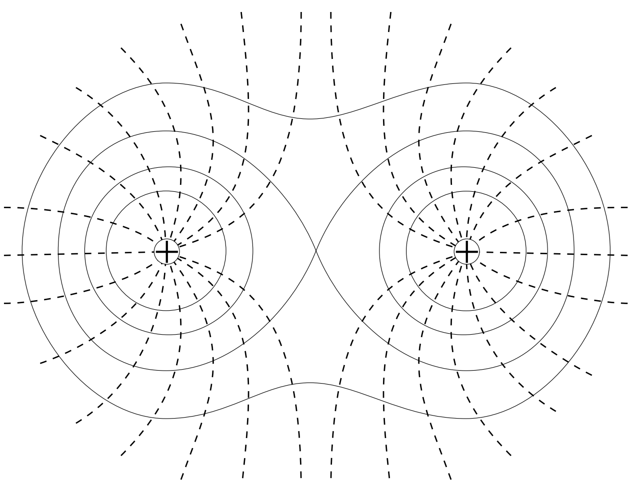
PHYSNET

MISN-0-420

ELECTRIC POTENTIAL



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ELECTRIC POTENTIAL

by

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CONTENTS

Title: **Electric Potential**

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Version: 4/30/2002 Evaluation: Stage 0

Length: 1 hr; 72 pages

Input Skills:

1. Vocabulary: electric field, electric field lines (MISN-0-419).
2. Write an expression for Coulomb potential energy (MISN-0-415).
3. Describe the electric field due to (a) a uniformly charged plane or (b) a uniformly charged sphere (MISN-0-419).

Output Skills (Knowledge):

K1. Vocabulary: electric potential, equipotential surface, potential drop, volt.

K2. State an expression for the electric potential due to a single charged particle.

K3. State the superposition principle for electric potentials due to several particles.

K4. State the relationship between electric field and potential drop.

K5. State how the electric field and equipotential surfaces are related.

Output Skills (Rule Application):

R1. Calculate the change in Coulomb potential energy of a charged particle which experiences a given change in electric potential.

R2. Given the change in electric potential between two specified points in a uniform electric field, determine the electric field.

Output Skills (Problem Solving):

S1. Given the charges and positions of particles near a point, determine the total electric potential at this point.

S2. Given a drawing of equipotentials: (a) describe the electric field or force on a particle in the field; (b) determine the work done on a particle as it moves between equipotentials.

MISN-0-420

ELECTRIC POTENTIAL

Abstract:
In the present unit we shall shift our attention from the Coulomb electric force (and corresponding electric field) to the potential energy associated with this force. Thus we shall achieve a simpler description of the electric interaction since it is easier to work with the potential energy (which is a number) than with the force (which is a vector). In addition, energy arguments are quite generally useful and provide the basis for many practical applications.

SECT.

A ELECTRIC POTENTIAL ENERGY AND POTENTIAL

Now: Go to tutorial section A

COULOMB ELECTRIC WORK AND POTENTIAL ENERGY

Consider the Coulomb electric force on a charged particle due to other charged particles at specified positions. This Coulomb force depends only on the relative positions of the interacting particles. Hence we know from Unit 415 that the work W_{AB} done on the charged particle moving from any point A to any other point B is independent of the process whereby it moves between these points. Thus we can simply write

$$W_{AB} = U_A - U_B \tag{A-1}$$

where the right side is the difference in the Coulomb electric potential energies of the particle at the points A and B. Here the potential energy U_A of the particle at the point A, due to its Coulomb interaction with all the other charged particles, is defined as

$$U_A = W_{AS} \tag{A-2}$$

the work done on the particle, by the Coulomb force due to the other charged particles, when the particle moves from the point A to some standard position S. (This standard position S is usually chosen to be so far away that the interaction of the particle with the other particles is negligibly small).

DEPENDENCE OF ELECTRIC POTENTIAL ENERGY ON CHARGE

Let us focus attention on some point P in the presence of charged particles at specified positions. (See Fig. A-1.) Suppose that a particle with charge q is placed at this point. In the preceding unit, we asked how the electric *force* on this particle depends on its charge q. Let us now ask how the electric *potential energy* U of this particle depends on its charge q.

► *U and magnitude of q*

We already know from the preceding unit that the electric force $\vec{F}$ on the particle is proportional to its charge q. For example, if the

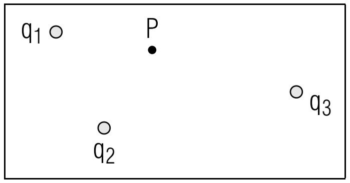


Fig. A-1: A point P near several charged particles.

charge q of the particle is 3 times as large, the electric force $\vec{F}$ on this particle is also 3 times as large. Hence the work done by this force on the particle moving from one specified point to another is also 3 times as large. Accordingly, the potential energy U of the particle at the point P is also 3 times as large [since this potential energy is, by its definition, Def. (A-2), merely equal to the work done on the particle when it moves from P to the standard position]. Thus the ratio U/q has the *same* value, irrespective of the magnitude of the charge q of the particle at P.

► *U and sign of q*

Similarly, if the charge q of the particle has the opposite sign, the electric force $\vec{F}$ on this particle has the opposite direction. Hence the work done by this force on the particle moving from one specified point to another would have the opposite sign. Accordingly, the potential energy U of the particle at the point P also has the opposite sign [since this potential energy is merely the work done on the particle when it moves from P to the standard position]. Thus the ratio U/q has the *same* value, irrespective of the *sign* of the charge q of the particle at P.

► *General conclusion*

Thus we arrive at this general conclusion: If a particle with charge q is placed at a point P, the potential energy U of this particle, due to its interaction with other charged particles at specified positions, is such that the ratio U/q is *independent* of the magnitude and sign of q. (Accordingly, the potential energy U of the particle is said to be proportional to its charge q.)

DEFINITION OF ELECTRIC POTENTIAL

The preceding conclusion can be summarized by writing

$$\boxed{\frac{U}{q} = V} \qquad \text{(A-3)}$$

where the quantity V is independent of the value of q. This quantity V is called the electric "potential" in accordance with this definition: *

* It is important to distinguish carefully between the different quantities "potential energy" and "potential."

Def. **Potential**: The potential V at a point P, due to charged particles at specified positions other than P, is the quantity $V = U/q$ where U is the electric potential energy, due to these charged particles, of *any* particle with charge q placed at P. (A-4)

Thus the electric potential V at a point is a quantity describing the relation between the electric potential energy and the charge of any particle at this point.

Although the potential $V = U/q$ at a point P does *not* depend on the charge q of the particular particle placed at P, it *does* ordinarily depend on the point P as well as on the charges and positions of all the other particles responsible for the potential energy U.

► *Unit of V*

According to Def. (A-4), the potential is, like the potential energy, a number and has this unit:

$$\boxed{\text{unit of } V = \frac{\text{joule}}{\text{coulomb}} = \text{volt}} \qquad \text{(A-5)}$$

where the unit "volt" (abbreviated as "V" is just a convenient abbreviation for J/C. [This unit is named in honor of Alessandro Volta (1745-1827), the inventor of the battery.]

► *Calculation of V*

The potential V at a point P is, by its definition $V = U/q$, the potential energy per unit charge (i.e., the potential energy divided by the corresponding charge) of *any* particle placed at P. Thus the potential V at a point P can be calculated as follows: (1) Imagine that *any* convenient particle (e.g., a particle with a charge of +1 coulomb) is placed at P. (2) Find the electric potential energy U of this particle due its interaction with all other charged particles at their specified positions. (3) Calculate the ratio $V = U/q$.

► *Utility of V*

Suppose that we know the potential V at a point P (e.g., from information about the electric potential energy of some particular charged particle at P). Then we can use this knowledge of V to find immediately the electric potential energy U of *any* particle, with whatever charge q, placed at P. Indeed, by Def. (A-4), the ratio U/q for any such particle must just be equal to the potential V. Hence $U/q = V$ or

$$U = qV \qquad \text{(A-6)}$$

Thus the electric potential energy of any particle at a point P (due to other charged particles at specified positions) is simply found by multiplying the charge q of this particle by the potential V at P. (Note that the electric potential energy U has the same sign as the potential V if the charge q of the particle is positive, but has the opposite sign if q is negative.)

► *Work and potential drop*

By Eq. (A-1), the electric work W_{AB} done on a particle moving from a point A to a point B is simply $W_{AB} = U_A - U_B$. In other words, this work is equal to the "drop" of the particle's electric potential energy. (We use the word "drop" of a quantity to mean the *initial* value minus the final value of this quantity.) *

> * The "drop" of a quantity and the "change" of a quantity are both differences, but the "drop" means the *initial* value minus the *final* value, while the "change" means the *final* value minus the *initial* value. Thus the drop has the same magnitude, but the opposite sign, as the change.

Suppose now that we know the potentials V_A and V_B at the points A and B. If the particle has a charge q, its electric potential energies at A and B are then $U_A = qV_A$ and $U_B = qV_B$. Hence the relation (A-1) between work and potential energy is equivalent to

$$W_{AB} = qV_A - qV_B = q(V_A - V_B) \qquad \text{(A-7)}$$

In other words, the electric work done on the particle moving from A to B is simply obtained by multiplying the charge q of the particle by the potential drop $(V_A - V_B)$ from the initial to the final position of the particle.

Example A-1: Work done on an electron in a battery

A battery is a device which uses chemical reactions to produce different potentials between two points called the "terminals" of the battery. These terminals are ordinarily points on two different metals (such as lead and zinc) immersed in an ionic solution (such as dilute sulphuric acid). As a result of chemical reactions within the battery, electrons and ions move between these metals so as to make one of them positively and the

other one negatively charged. Consequently, the terminals of the battery acquire potentials differing typically by about 1 volt.

Suppose that the potentials V_A and V_B of the terminals A and B of a battery differ by 1.00 volt. What then is the magnitude $|W_{AB}|$ of the electric work done on an electron moving between the terminals of this battery? The charge q of an electron has a magnitude $e \approx 1.60 \times 10^{-19}$ coulomb. According to Eq. (A-7), the magnitude of the work done on such an electron moving between the terminals A and B is then

$$|W_{AB}| = e|V_A - V_B| = (1.60 \times 10^{-19}\,\text{coulomb})(1.00\,\text{volt})$$

or

$$|W_{AB}| = 1.60 \times 10^{-19}\,\text{joule} \qquad \text{(A-8)}$$

In such a battery, the chemical energy released in a reaction between two molecules provides the work necessary to move an electron from one terminal to the other. Hence the work found in Eq. (A-8) represents a typical magnitude of the chemical energy released in a reaction between two molecules. Since this chemical energy is very small compared to a joule, it is useful to introduce a special unit of energy particularly convenient for expressing chemical energies. This unit of energy, called the "electron volt" (abbreviated "eV") is defined to be equal to the magnitude of the energy required to move an electron (or any other particle with charge of magnitude e) between two points having potentials differing by exactly 1 volt. According to Eq. (A-8), the "electron volt" is thus approximately equal to

$$1\,\text{electron volt} \approx 1.6 \times 10^{-19}\,\text{joule} \qquad \text{(A-9)}$$

For example, the magnitude of the energy required to move an electron between two points having potentials differing by 3 volt is then simply 3 electron volt.

Now: Go to tutorial frame [a-8].

Understanding the Definition of Potential (Cap. 1a)

$\boxed{\text{A-1}}$ *Relating quantities:* The potential at a point P very near a battery terminal is 1.5 volt. (a) If a particle of charge -2×10^{-10} C is suspended at P by a thread, what is the potential energy of this particle? (b) Now suppose a new particle of charge 6×10^{-10} C is suspended at P. Is this particle's electric potential energy different from the value of U found in part (a)? If so, what is it? Is the potential at P different from the value given in part (a)? If so, what is it? *(Answer: 4) ([a-8], [a-9], [a-10])*

Relating Work to Potential Drop (Cap. 3)

A-2 A "flashlight" battery inserted in a portable tape recorder has a potential drop of 1.5 V from terminal A to terminal B. (a) What is the potential drop from the terminal B to the terminal A of this battery? (b) What is the work done on an electron (of charge -1.6×10^{-19} C) as it moves from B to A along each of these paths: the path through the motor driving the tape, and the path through the small light indicating that the tape recorder is turned on? Express your answers both in terms of the unit joule and in terms of the unit electron volt. *(Answer: 1) ([a-11], [a-12])*

A-3 High speed protons are useful for various purposes, including sterilizing food through irradiation. Such particles can be produced in the following way by a "tandem" Van-de-Graaff accelerator. A negatively charged hydrogen ion (a proton with two electrons) accelerates towards a positively charged "terminal," moving from the point A to the point B in Fig. A-2. Then the ion passes through the cylindrical "terminal" and through a gas-filled tube in which the two electrons are lost so that a positively charged proton emerges. This particle then accelerates away from the terminal, moving from the point C to the point D. For a typical accelerator, the potentials at A and D are 0 volt, while the potentials of the points B and C on the terminal are 1×10^6 volt. Because the mass of an electron is so much less than the mass of a proton, we can consider the ion and proton as a single particle with unchanging mass, but with a charge which changes from $-e$ to $+e$ as it passes through the gas-filled tube. (e = magnitude of the electron's charge.) Use the unit electron volt to express values for the following works done by the charged "terminal" on this accelerating particle: (a) Work done as it moves from A to B. (b) Work done as it moves from B to C. (c) Work done as it moves from C to D. (d) Total work done on the particle as it moves from A to D. (e) If the particle is initially essentially at rest, and it interacts appreciably only with the accelerator, what is its final kinetic energy? *(Answer: 7) ([a-11], [a-12])*

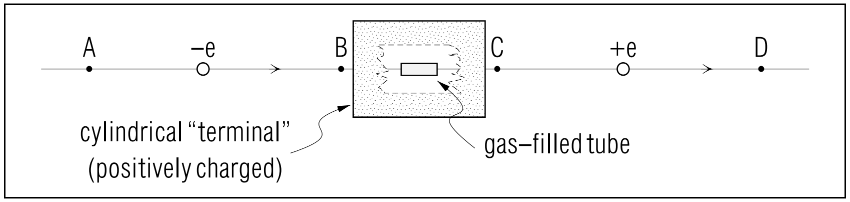


Fig. A-2.

SECT.

B POTENTIAL PRODUCED BY CHARGED PARTICLES

Now: Go to tutorial section B.

The potential produced at any point by one or more charged particles can be readily found from a knowledge of the electric potential energy of interaction between charged particles.

POTENTIAL DUE TO A SINGLE CHARGED PARTICLE

Consider a single particle 1 with charge q_1. What then is the potential V at a point P at a distance R from this charged particle? (See Fig. B-1.)

► *Potential energy*

To find the potential at the point P, we need only imagine that a particle with any convenient charge q is placed at the point P. The electric potential energy U of this particle (due to its interaction with particle 1) is then equal to the work done by the Coulomb electric force when the particle moves from P to a standard position very far from particle 1. As we already found in text section D of Unit 415, this potential energy U is simply

$$U = k_e \frac{q q_1}{R} \tag{B-1}$$

Note that this potential energy U is proportional to $(1/R)$, unlike the Coulomb force which is proportional to $(1/R^2)$, in agreement with the fact that the potential energy has the units of work (i.e., of a force *multiplied by a distance*). The potential energy U is properly positive if the electric force on the particle with charge q due to the particle with charge q_1 is repulsive (i.e., if both charges have the same sign). Conversely, U is negative if the electric force is attractive (i.e., if both charges have opposite signs).

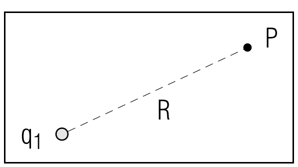


Fig. B-1: Point P near a single particle.

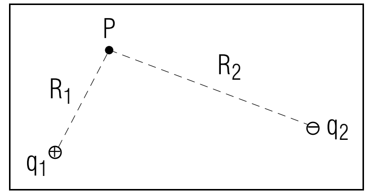


Fig. B-2: Point P near two charged particles.

► *Potential*

To find the potential at the point P, we need only use the definition $V = U/q$. Dividing Eq. (B-1) by q, we thus obtain

$$\boxed{V = k_e \frac{q_1}{R}} \tag{B-2}$$

Note that the potential produced at a point P by the particle with charge q_1 is proportional to q_1 and inversely proportional to the distance R of P from this charged particle (unlike the electric field at P which is inversely proportional to R^2). The sign of the potential V is seen to be positive if the charge q_1 is positive, and negative if the sign of q_1 is negative.

All the important relations we have discussed are summarized in Table B-1.

$F = k_e \dfrac{\lvert q\rvert \lvert q_1\rvert}{R^2}$	$U = k_e \dfrac{q q_1}{R}$
$E = k_e \dfrac{\lvert q_1\rvert}{R^2}$	$V = k_e \dfrac{q_1}{R}$

Table B-1: Summary of results derived from Coulomb's law. Magnitude of Coulomb force $\vec{F}$ and potential energy U between two interacting particles. Magnitude of electric field $\vec{E}$ and potential V at a distance R from a charged particle.

POTENTIAL DUE TO ANY NUMBER OF CHARGED PARTICLES

Consider several charged particles, e.g., two particles 1 and 2 with charges q_1 and q_2, located at specified positions. What then is the potential at some point P due to these particles? (See Fig. B-2.)

► *Superposition of potentials*

Imagine that a particle with any convenient charge q is placed at the point P. Because of the superposition principle for the forces acting on this particle, there is then a corresponding superposition principle for the work done by these forces, and thus also for the potential energy U due to these forces. (See the discussion in text section D of Unit 415.) Accordingly:

$$U = U_1 + U_2 \tag{B-3}$$

where U_1 is the potential energy of the particle at P due to its interaction with particle 1 alone, and U_2 is the potential energy of the particle at P due to its interaction with particle 2 alone. By dividing both sides of Eq. (B-3) by the charge q, we then find correspondingly that the potential $V = U/q$ is

$$V = V_1 + V_2 \tag{B-4}$$

where $V_1 = U_1/q$ is the potential at P due to particle 1 alone and $V_2 = U_2/q$ is the potential at P due to particle 2 alone. Thus we arrive at this general conclusion:

Superposition principle for potentials: The potential at a point due to any number of charged particles is the sum of the potentials produced at this point by all these particles separately.	(B-5)

Since the potential is a *number*, the sum of the potentials in Rule (B-5) is a simple *numerical* sum. Hence the superposition principle for potentials is much simpler than the superposition principle for electric fields, statement (B-5) of Unit 419, since that principle involves a *vector* sum.

By Eq. (B-2), we already know how to find the potential produced at a point P by any single charged particle separately. Hence Rule (B-5) tells us that we need merely to add these potentials numerically in order to find the potential produced at P by any number of charged particles.

Example B-1: Calculation of potential due to two particles

In Fig. B-2, suppose that the particle 1 has a charge $q_1 = 4.0 \times 10^{-9}$ C and is at a distance $R_1 = 0.20$ m from the point P, while the particle 2 has a charge $q_2 = -5.0 \times 10^{-9}$ C and is at a distance $R_2 = 0.30$ m from P. What then is the potential V at the point P due to both of these particles?

The potential V_1 due to particle 1 alone is, by Eq. (B-2),

$$V_1 = k_e \frac{q_1}{R_1} = (9.0 \times 10^9\,\mathrm{N\,m^2/C^2}) \frac{4.0 \times 10^{-9}\,\mathrm{C}}{0.20\,\mathrm{m}} = 180\,\mathrm{volt}$$

Similarly the potential V_2 due to particle 2 alone is

$$V_2 = k_e \frac{q_2}{R_2} = (9.0 \times 10^9\,\mathrm{N\,m^2/C^2}) \frac{-5.0 \times 10^{-9}\,\mathrm{C}}{0.30\,\mathrm{m}} = -150\,\mathrm{volt}$$

Hence

$$V = V_1 + V_2 = 180\,\mathrm{volt} + (-150\,\mathrm{volt}) = 30\,\mathrm{volt}$$

Understanding Potential due to a Charged Particle (Cap. 1b)

B-1 *Statement and example:* Two small plastic balls with charges $q_1 = 2.0 \times 10^{-8}$ C and $q_2 = -3.0 \times 10^{-8}$ C are suspended from threads so that they are separated by a distance of $R = 3.0$ cm. (a) Use the algebraic symbols provided to write an expression for the electric potential due to ball 1 at the location of ball 2. Then find the value of this potential. (b) *Comparing potential and potential energy.* What is the potential energy of ball 2 due to ball 1? Find both the potential at the location of ball 1 due to ball 2, and the potential energy of ball 1 due to ball 2. *(Answer: 5) (Suggestion: [s-4])*

B-2 *Comparing relations describing two-particle interactions:* (a) Two particles of charge q_1 and q_2 are located at points A and B a distance R apart. Use these algebraic symbols to write expressions for these quantities: The electric potential V_B and the magnitude E_B of the electric field produced at B by the particle at A. The potential energy U_B of the particle at B due to the particle at A. The magnitude F_{BA} of the force on the particle at B due to the particle at A. (b) What is the simplest SI unit for each of the quantities V_B, E_B, U_B, and F_{BA}? *(Answer: 9)*

Describing Potential due to Any Number of Particles (Cap. 2)

B-3 Figure B-3 shows two particles each with charge $q = 2.0 \times 10^{-8}$ C. For the points A and B shown on the diagram, find the electric field and potential due to these two particles. Which of these quantities is easier to find? *(Answer: 12) (Suggestion: [s-10])*

B-4 Four particles, each of charge q, are located at the corners of a square of side L. Use the preceding symbols to express the electric field and the electric potential at the center of this square. *(Answer: 2) (Practice: [p-1])*

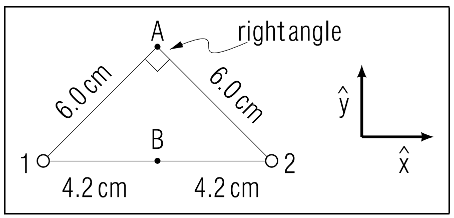


Fig. B-3.

SECT.

C RELATION BETWEEN ELECTRIC POTENTIAL AND FIELD

► *Work and potential energy*

According to the fundamental relation (A-1)

$$W_{AB} = U_A - U_B \tag{C-1}$$

i.e., the electric work done on a particle is equal to the drop in its electric potential energy. Suppose then that the displacement D of the particle from A to B is small enough so that the electric force $\vec{F}$ on the particle is constant during this displacement. Then the definition of work, discussed in Unit 414, implies that $W_{AB} = FD_F$ where D_F is the numerical component of the displacement along the force. Thus Eq. (A-1) can be written as

$$FD_F = U_A - U_B \tag{C-2}$$

► *Work and potential*

To find corresponding relations between the electric fieldand potential, we need only divide the preceding relations by the charge q of the particle. Thus Eq. (A-1) becomes

$$\frac{W_{AB}}{q} = \frac{U_A}{q} - \frac{U_B}{q} = V_A - V_B \tag{C-3}$$

This states that the electric work *per unit charge* (i.e., the work divided by the corresponding charge) done on a particle moving from a point A to a point B is equal to the potential drop from A to B.

► *Work and field*

But the work per unit charge in a small enough displacement is

$$\frac{W_{AB}}{q} = \frac{F}{q} D_F = ED_E \tag{C-4}$$

The last expression on the right is certainly correct if the charge q is positive. For then F/q is just the magnitude E of the electric field acting on the particle, and the component D_F along the electric force $\vec{F}$ is the same as that along the electric field $\vec{E}$ since $\vec{F}$ has the same direction as $\vec{E}$. (The expression is also correct if q is negative.) *

> * The *magnitude* E of the field is a positive quantity, as is the *magnitude* of any number. That is, $E = |F/q|$ so E is independent of the sign of q. If q is negative, the electric force $\vec{F}$ has a direction *opposite* to that of the electric field $\vec{E}$. The components of $\vec{D}$ along these vectors have opposite signs so that $D_E = -D_F$. Hence $(F/q)D_F = (-E)(-D_E) = ED_E$, the same result as for a positive charge q.

The relation (C-4) merely states that the electric work per unit charge is obtained by multiplying the magnitude of the electric field (i.e , the electric force per unit charge) by the component of the particle's displacement along the field.

► *Field and potential*

The result Eq. (C-3) relating the work per unit charge to the potential drop can then be written as

$$ED_E = V_A - V_B \tag{C-5}$$

where the displacement $\vec{D}$ is assumed to be small enough so that the electric field $\vec{E}$ remains constant during this displacement. All these relations are compactly summarized in Table C-1. In particular, Eq. (C-5), which relates the work per unit charge ED_E and the potential drop, provides a very useful connection between the electric field and the potential.

$$FD_F = W_{AB} = U_A - U_B$$

$$E = \frac{F}{q} \qquad V = \frac{U}{q}$$

$$ED_E = \frac{W_{AB}}{q} = V_A - V_B$$

Table C-1: Summary of important relations connecting force (or field), work, and potential energy (or potential). The displacement $\vec{D}$ is assumed to be small.

POTENTIAL AND FIELD NEAR A POINT

Consider a point A at which there is an electric field $\vec{E}$ due to charged particles at other positions. How then is the potential V_B at any point B in the vicinity of A related to the potential V_A at A? (See Fig. C-1.) To answer this question, we can use the relation (C-4) and consider various

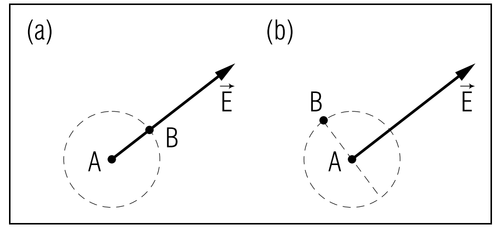


Fig. C-1: Potential at several points B near a fixed point A.

points B all of which are at the same distance D from A (this distance being small enough so that the electric field does not change appreciably between A and any of these points).

► *Displacement along* $\vec{E}$

Consider first a neighboring point B such that the displacement from A to B is *along* the direction of the electric field at A. (See Fig. C-1a). Then the component D_E of the displacement along the field has its maximum possible value D. Hence the electric work per unit charge done on a particle moving from A to B has the maximum possible value ED. According to Eq. (C-4), the potential drop $V_A - V_B$ is then also maximum. Hence the potential V_B at B is *smaller* than the potential V_A at A by the maximum possible amount. *

> * Similarly, if the displacement from A to B is *opposite* to the electric field, V_B is *larger* than V_A by the largest possible amount.

► *Displacement perpendicular to* $\vec{E}$

Consider now a point B such that the displacement from A to B is *perpendicular* to the direction of the electric field at A. (See Fig. C-1b.) Then the component of this displacement along the field is zero, so that the electric work per unit charge done on a particle moving from A to B is also zero. According to Eq. (C-4), the potential drop $V_A - V_B$ is then also zero. Hence the potential V_B at B is the *same* as the potential V_A at A.

► *Summary*

The preceding comments lead to this conclusion:

> In the neighborhood of any point, the potential decreases maximally along the direction of the electric field and remains unchanged along any direction perpendicular to the field. (C-6)

► *Finding field from potentials*
We have seen how to use the relation (C-5) to find potentials from information about the electric field. Conversely, this relation can also be used to find the electric field from information about potentials. For example, suppose that we know the potential at all points in the vicinity of some point A. Then this information is sufficient to find the electric field $\vec{E}$ at A. Indeed, we need only compare the potential at A with that at various other points at some small fixed distance D from A. Then the *direction* of $\vec{E}$ is simply that along which the potential drop from A is maximum. Furthermore, the *magnitude* of $\vec{E}$ is such that ED is simply equal to this potential drop.

It is often much simpler to find values of the potential than values of the electric field, since the calculation of the potential involves merely numbers while the calculation of the field involves vectors. The conclusion of the preceding paragraph is thus very useful since it allows one to find electric fields from a simpler calculation of potentials.

CONVENTIONAL UNIT OF ELECTRIC FIELD

According to Eq. (C-5), $E = (V_A - V_B)/D_E$. Hence

$$\text{unit of } E = \frac{\text{volt}}{\text{meter}} \tag{C-7}$$

This is, in fact, the commonly used SI unit of electric field since it relates the field directly to the potential, the *numerical* quantity measured most simply in experiments. Note that the right side of Eq. (C-7) is equivalent to

$$\frac{\text{volt}}{\text{meter}} = \frac{\text{joule}}{\text{coulomb(meter)}} = \frac{(\text{newton})(\text{meter})}{(\text{coulomb})(\text{meter})} = \frac{\text{newton}}{\text{coulomb}} \tag{C-8}$$

the unit of electric field which we have used previously.

POTENTIAL DUE TO A UNIFORMLY CHARGED SPHERE

To illustrate how information about the electric field can be immediately used to infer corresponding information about the potential, let

us find the potential due to a uniformly charged sphere (or spherical surface). As discussed in text section F of Unit 419, at any point on or outside such a sphere the electric field is the same as if the entire charge Q of the sphere were concentrated at its center. Since the potential V at a point on or outside such a uniformly charged sphere depends only on the electric field outside this sphere, this potential must then also be the same as if the entire charge of the sphere were concentrated at its center. According to Eq. (B-2), the potential V at a point at a distance R from the center of the sphere (where R is equal to or larger than the radius of the sphere) is thus simply equal to $V = k_e Q/R$.

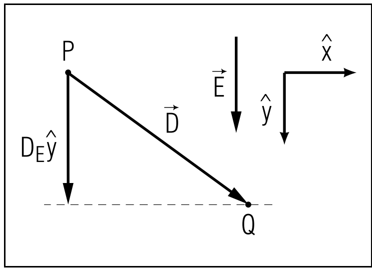


Fig. C-2.

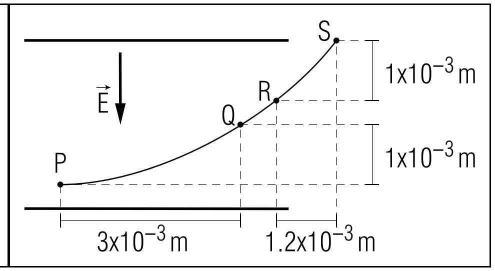


Fig. C-3.

Understanding the Relation between Potential and Field (Cap. 1c)

C-1 *Statement and example:* (a) In a region between the charged parallel plates in an oscilloscope, the electric field $\vec{E}$ is uniform. Fig. C-2 shows two points and in this region, the displacement $\vec{D}$ from P to Q, and the component vector $D_E\hat{y}$ parallel to the direction of $\vec{E}$. Use the preceding symbols to write an expression for the potential drop $V_P - V_Q$ from the point P to the point Q. (b) Does the potential V increase or decrease from P to Q? (c) If $|\vec{E}| = 3 \times 10^4$ N/C and $|D_E| = 0.01$ m, what is the potential drop $V_P - V_Q$ in terms of the unit volt? *(Answer: 8) (Suggestion: [s-6])*

C-2 *Interpretation of D_E:* (a) Fig. C-3 shows the path of an electron moving between the parallel charged deflecting plates of an oscilloscope. The points P and Q are in a region in which the uniform electric field has a magnitude of 2×10^4 N/C. What is the potential drop from the point P to the point Q? (b) *Applicability:* As the electron travels between the points R and S shown in Fig. C-3, it moves through a region in which the electric field $\vec{E}$ is not uniform. Briefly describe why the relation $V_R - V_S = ED_E$ can not be used directly to find the potential drop from R to S. *(Answer: 13)*

More practice for this Capability: [p-3], [p-4]

Relating Quantities Describing the Electric Interaction (Cap. 3)

The following problems require applying appropriately and without confusion the relations summarized in Table C-1.

C-3 Each point on a uniformly charged metal plate A has a potential $V_A = 0$, and each point on a parallel plate B at a distance of 0.50 cm from A has a potential $V_B = 200$ volt. (a) What is the value of the uniform electric field in the region between these plates? (b) What is the magnitude of the electric force on a proton (with charge 1.6×10^{-19} C) located between these plates? (c) Use $W_{AB} = FD_F$ to find the electric work done on this proton as it moves from a point on plate A to a point on plate B. *(Answer: 3) (Suggestion: [s-9])*

C-4 (a) In the preceding problem, what is the potential drop from a point on plate A to a point on plate B? (b) What is the drop $U_A - U_B$ in the potential energy of the proton as it moves from plate A to plate B? (c) What is the corresponding work done on this proton? Does this answer check with your answer to part (c) of C-3? *(Answer: 10)*

C-5 A uniformly charged sphere in a Van-de-Graaff accelerator has a radius of 0.090 m and a charge of 2.0×10^{-6} C. (a) What is the potential drop from a point A at the surface of this sphere to a point B very far from this sphere? (b) What is the work done by the sphere on an alpha particle with charge 3.2×10^{-19} C as it moves from A to B? (c) *Review:* If the alpha particle moves from A to B under the sole influence of the Van-de-Graaff sphere, does its speed increase or decrease? *(Answer: 6) (Suggestion: [s-8])*

More practice for this Capability: [p-5], [p-6]

Describing Potential due to a Sphere

C-6 (a) What is the electric potential of a point A at the surface of a uniformly charged sphere of radius 0.1 m in a Van-de-Graaff accelerator if this sphere has a total charge of 3.0×10^{-6} C? What is the potential energy of an alpha particle of charge 3.2×10^{-19} C located at A? (b) *Review:* Under the sole influence of the Van-de-Graaff sphere, the alpha particle moves from A to a point S very far from the sphere. What is the work W_{AS} done on the alpha particle? *(Answer: 11) (Practice: [p-2])*

SECT.

D EQUIPOTENTIALS

In the preceding unit we found that we could easily visualize the electric field by means of electric field lines. Let us now try to find some equally convenient pictorial way to help us visualize the electric *potential* due to charged particles.

► *Equipotential surface*

Consider some point A where the potential due to various charged particles is equal to V. Then we know from the preceding section that the potential has also the same value V at all neighboring points lying along directions perpendicular to the electric field $\vec{E}$ at A, i.e., along all points lying on a very small plane through A and perpendicular to $\vec{E}$. (See Fig. D-1a.) Consider now some other point B near the edge of this small plane. By repeating the preceding argument, we conclude that the potential has also the same value V as at B (or A) at all those points on a very small plane through B and perpendicular to the electric field at B. By continuing in this fashion, we can thus successively generate a whole collection of very small overlapping planes which (somewhat like the scales of a fish) constitute a continuous surface all of whose points have the same potential V, (See Fig. D-1b.) Such a surface is called an "equipotential surface" in accordance with this definition:

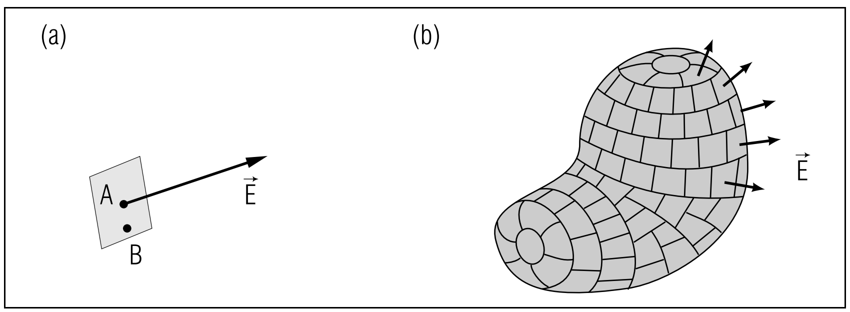


Fig. D-1: Equipotential surfaces. (a) Small plane all of whose points have the same potential as point A. (b) Entire equipotential surface generated by many such small planes.

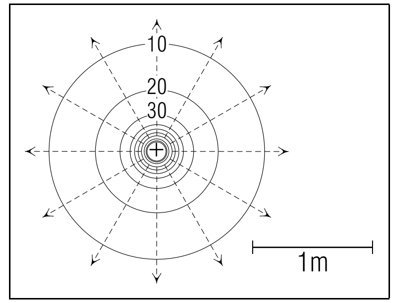


Fig. D-2: Equipotential lines (solid lines) and electric field lines (dashed lines) due to a particle with a charge of 10^{-9} coulomb. The potentials, calculated from $V = k_e q/R$, are expressed in volt.

Def. **Equipotential surface**: A surface all of whose points have the same potential. * (D-1)

* In the special case where the electric field $\vec{E}$ at a point is zero, *all* points near this point have the same potential. Hence there exists then an equipotential region rather than merely an equipotential surface.

► *Equipotential line*

Note that the intersection of an equipotential surface with a plane (or other surface) results in an "equipotential line," i.e., in a line all of whose points have the same potential.

► *Applications*

The electric potential in a certain region of space can then be conveniently visualized by imagining a set of equipotential surfaces each of which corresponds to a somewhat different value of the potential. In particular, the potential in some plane (e.g., the plane of a piece of paper) can be very simply visualized by drawing a set of equipotential lines (intersections of equipotential surfaces with the plane), each of which corresponds to a somewhat different value of the potential. For example, Fig. D-2 shows the equipotential lines due to a single positively charged particle. The equipotential surfaces are in this case simply spheres (and the equipotential lines correspondingly just circles) since the potential at a fixed distance from a single charged particle has the same value in all directions from this particle.

► *Analogies*

We have seen how the electric potential can be readily visualized by drawing a set of surfaces (or lines) each of which corresponds to some

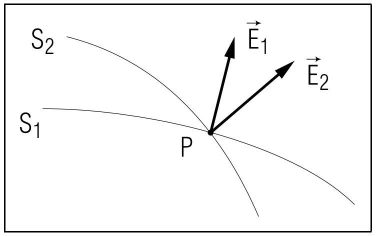


Fig. D-3: Hypothetical intersection of two equipotential surfaces S_1 and S_2.

different fixed value of the potential. The same method is commonly used to visualize other numerical quantities. For example, the "topographic" maps used for hiking provide information about the height of the terrain by indicating a set of lines ("contour lines") each of which corresponds to some different fixed height above sea level. Similarly, weather maps commonly provide information about atmospheric pressure at various locations by indicating a set of lines ("isobars") each of which corresponds to some different fixed value of the atmospheric pressure.

PROPERTIES OF EQUIPOTENTIAL SURFACES

► *$\vec{E} \perp$ to surface*

As discussed earlier, the electric field at any point of an equipotential surface is perpendicular to the portion of surface near this point. (In other words, the electric field can have no component *along* the surface since the potential does not change along the surface.) Hence we can state this important property of equipotential surfaces:

The electric field at any point of an equipotential surface must be perpendicular to the surface at this point.	(D-2)

► *No intersection*

Can two equipotential surfaces ever intersect (i.e., cross each other)? To answer this question, suppose that two such surfaces S_1 and S_2 did intersect at some point P, as indicated schematically in Fig. D-3. Then the electric field at P should have a direction perpendicular to the surface S_1 (as indicated by $\vec{E}_1$) and should also have a direction perpendicular to S_2 (as indicated by $\vec{E}_2$). But these conclusions are inconsistent with the fact that the electric field $\vec{E}$ at any point has a unique direction (unless $\vec{E} = 0$ so that $\vec{E}$ has no direction). Accordingly we see that two equipotential surfaces *cannot* intersect, except at points where the electric field is zero.

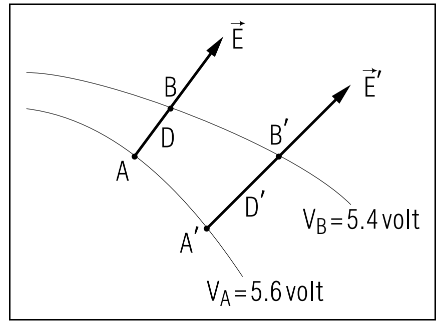


Fig. D-4: Two neighboring equipotential surfaces corresponding to potentials having values V_A and V_B. [If $D = 0.005\,\text{m}$, the magnitude of the electric field at A is $(0.2\,\text{V})/(0.005\,\text{m}) = 40\,\text{V/m}$.]

► *Finding $\vec{E}$ from equipotentials*

Our discussion in Sec. C implies that a knowledge of the equipotential surfaces in the vicinity of any point A should allow us to find the electric field $\vec{E}$ at A. To show this, we need only look at Fig. D-4 and note that:

(1) The *direction* of $\vec{E}$ is perpendicular to the equipotential surface at A and points from A toward the neighboring equipotential surface of *lower* potential (since motion along the electric field should result in a positive potential drop).

(2) The *magnitude* of $\vec{E}$ can be found by considering the displacement $\vec{D}$ (perpendicular to the equipotential surface at A) from A to a point B on the neighboring equipotential surface of lower potential. By Eq. (C-5), the work per unit charge along this displacement must be equal to the potential drop. Hence $ED = V_A - V_B$ or

$$E = \frac{V_A - V_B}{D} \qquad \text{(D-3)}$$

Thus the magnitude E of the field at the point A can be found from the known potentials V_A and V_B of the neighboring equipotential surfaces and from the separation D of these surfaces near the point A. *

* Information about equipotential *lines* in a plane allows one similarly to obtain information about the *component vector* of the electric field parallel to this plane.

► *E and equipotential separation*

The potential drop between the same two equipotential surfaces is always the same, but the separation D between them may vary from

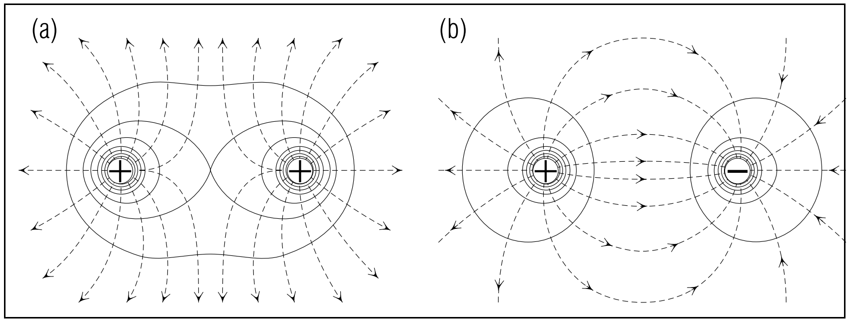


Fig. D-5: Equipotential surfaces (solid lines) and field lines (dashed lines) due to two particles with charges of equal magnitude. (a) Charges of like signs. (b) Charges of unlike signs.

point to point. At a point (such as A' in Fig. D-4) where the separation between the equipotential surfaces is larger, the magnitude of the electric field is then, according to Eq. (D-3), *smaller*.

The magnitude of the electric field is larger where the separation between neighboring equipotential surfaces is smaller.	(D-4)

This conclusion makes sense since the potential drop (or work per unit charge) between the two equipotential surfaces is everywhere the same. Hence places where the *distance* between the surfaces is larger must be places where the *force* per unit charge (or field) is smaller.

EXAMPLES OF EQUIPOTENTIALS

Let us now look at various examples of equipotential surfaces (or, more precisely, of the equipotential lines which are the intersections of these surfaces with the plane of the page).

► *Two charged particles*

Figure D-5 shows the equipotentials and electric field lines due to two particles having charges of the same magnitude. Note that the electric field lines are everywhere properly perpendicular to these equipotential surfaces. The potential V at every point is, of course, just the sum $V = V_1 + V_2$ of the potentials due to each of the particles. (Very close to each particle the potential due to this particle is much larger than that due to the other one. Hence the equipotential surfaces very close to

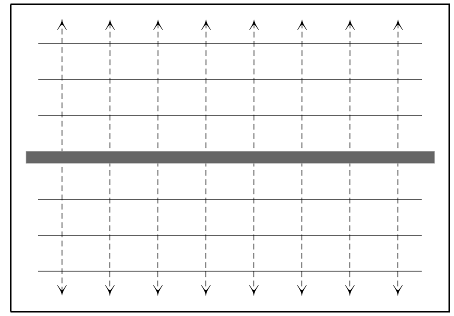


Fig. D-6: Equipotential surfaces (solid lines) and field lines (dashed lines) in the vicinity of a uniformly positively charged plane.

a particle resemble the spherical equipotential surfaces due this single particle alone.)

► *Charged plane*

In the vicinity of a uniformly charged plane, the electric field is perpendicular to the plane and of constant magnitude (as discussed in Unit 419). Hence the equipotential surfaces perpendicular to the corresponding field lines are merely uniformly spaced planes, as shown in Fig. D-6.

► *Human heart*

The biochemical processes responsible for the contraction of the heart muscle are accompanied by the motion of charged particles in the nerves and muscles of the heart. These charged particles produce a measurable potential near the heart and on the surface of the body. Fig. D-8 shows, at a particular instant during the heart cycle, the equipotential lines produced on a person's chest by these charged particles. (In medical practice, the "electrocardiogram" is obtained from measurements of these potentials on a person's skin and is then used to make important diagnostic inferences about the normal or abnormal functioning of the heart.)

Using Equipotentials to Describe Forces and Fields (Cap. 4a)

D-1 Figure D-7 shows the equipotential lines in the plane of two straight wires connected to the terminals of a flashlight battery. The equipotential lines shown near the point P are separated by about 0.10 cm, and the electric field $\vec{E}$ at P lies in the plane of the paper. (a) What is the value of $\vec{E}$? (b) What is the electric force on an ionized hydrogen molecule (H_2^+ with charge 1.6×10^{-19} C) located at the point

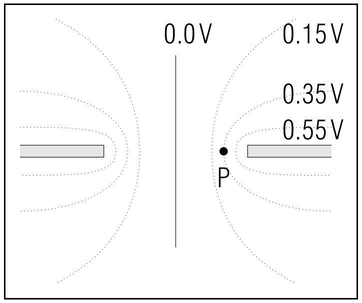


Fig. D-7.

P? (Answer: 14)

D-2 The equipotential lines near the point A shown in Fig. D-8 are separated by about 1.5 cm. From these equipotential lines, one can find the component in the plane of the man's chest of the electric field due to the heart at the instant shown. (a) What is the magnitude of this component field? Describe the direction of this component field. (b) What is the component along the skin surface of the corresponding electric force on a Cl^- ion (of charge -1.6×10^{-19} C) located just under the skin of the chest at the point A? *(Answer: 19) (Practice: [p-7])*

Using Equipotentials to Find Work (Cap. 4b)

D-3 Figure D-9 shows the path of an electron as it moves from a point A to a point B near the edges of two parallel charged deflecting plates in an oscilloscope tube. The labeled lines are equipotential lines

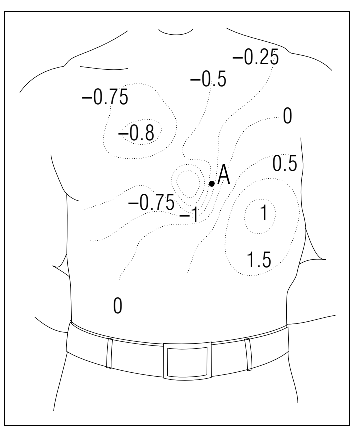


Fig. D-8: Equipotential lines on the chest of a person at a particular instant during the heart cycle. The numbers indicate potentials expressed in millivolts. Pt. A is midway between the −0.5 and the −0.25 lines. [Figure adapted from B. Taccardi, Circulation Research, vol. 12, p. 341 (1963)].

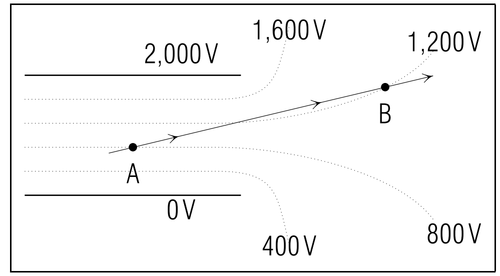


Fig. D-9.

having the indicated values of potential. What is the electric work W_{AB} done on the electron (of charge -1.6×10^{-19} C) as it moves from A to B? *(Answer: 17) (Suggestion: [s-1])*

D-4 Suppose that a particle P with charge 1×10^{-8} C moves from a point A very far from the charged particle shown in Fig. D-2 to a point B located on the equipotential surface labeled by 30. What is the electric work W_{AB} done on the particle P? *(Answer: 22) (Suggestion: [s-1]) (Practice: [p-8])*

Knowing About Characteristics of Equipotentials

D-5 Which of the diagrams in Fig. D-10 best describes the electric field lines (solid) and equipotential lines (dotted) due to two particles of charge $+4 \times 10^{-8}$ C and -3×10^{-8} C? *(Answer: 25)*

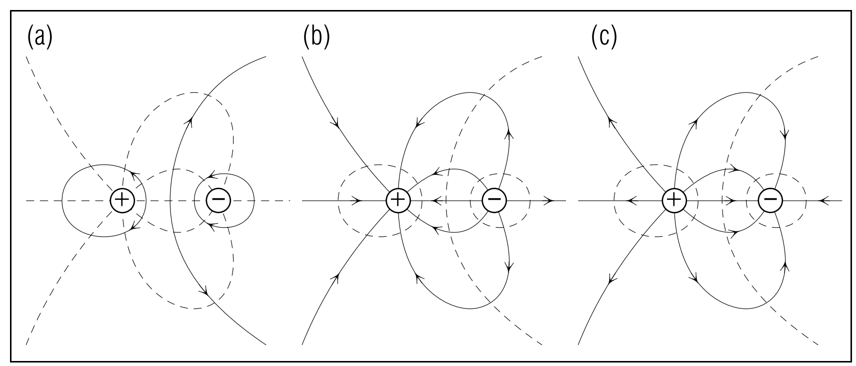


Fig. D-10.

SECT.

E SUMMARY

DEFINITIONS

electric potential; Eq. (A-3), Def. (A-4)

volt; Eq. (A-5)

equipotential surface; Def. (D-1)

IMPORTANT RESULTS

Potential due to a single charged particle q: Eq. (B-2)

$V = k_e q/R$

Superposition principle for potentials due to several particles: Rule (B-5)

$V = V_1 + V_2 + \ldots$

Relation between field and potential drop: Eq. (C-4), Table C-1

$ED_E = V_A - V_B$ (for $\vec{D}$ small enough)

Field and equipotential surface: Rule (D-2)

$\vec{E}$ must be everywhere perpendicular to an equipotential surface.

NEW CAPABILITIES

(1) Understand these relations:

(a) The definition of potential, $V = U/q$ (Sec. A),

(b) The relation $V = k_e q/R$ describing the potential due to a charged particle or the potential outside of a spherically symmetric charge distribution (Sec. B),

(c) The relation $ED_E = V_A - V_B$ between electric field and electric potential (Sec. C, [p-3], [p-4]).

(2) For a given point, use the charges and positions of nearby particles to find the potential at this point (Sec. B, [p-1], [p-2]).

(3) Relate without confusion quantities in the relations listed in Table C-1 (Sects. A and C, [p-5], [p-6]).

(4) Use a drawing of equipotentials:

(a) To describe the electric field or force on a particle,

(b) To find the work done on a particle as it moves between equipotentials (Sec. D, [p-7], [p-8]).

Study aids are available in:

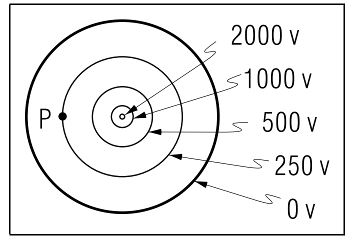


Fig. E-1.

Tutorial section A: Review of electric force, work, and potential energy.

Tutorial section B: Potential energy of one particle due to another.

Tutorial section E: Relating descriptions of electric interactions.

Tutorial section F: Additional problems on biological applications.

Relating Descriptions of the Electric Interaction (Cap. 3,4)

E-1 A geiger tube consists of a gas-filled charged metal tube with a wire of opposite charge along its axis. Fig. E-1 shows the equipotential lines in a plane perpendicular to the axis of this tube. When a high speed charged particle passes through the tube, it ionizes a gas molecule, producing a free electron at the point P. (a) What is the direction of the electric force on this electron? Under the influence of this force, does the electron ultimately move to the central wire, or to the outer tube? (b) What is the electric work done on the electron as it moves from P to a final position on the wire or tube? [Similar motion of many electrons and ions produces a small but detectable current each time a charged particle passes through the geiger tube.] *(Answer: 15)*

E-2 The average position of an electron in a hydrogen atom is about 0.5 Å from the proton (of charge $+e$). (a) What is the electric potential due to the proton at the average position of the electron? (b) What is the corresponding electric potential energy of an electron at this position? *(Answer: 21)*

Characteristics of Electric Potential and Field

E-3 (a) in a region of space where the electric field is zero, must the potential also be zero? Must the potential be constant? (b) In a region where the electric potential is zero, must the electric field be zero? Must this field be constant? *(Answer: 18) (Suggestion: [s-7])*

E-4 Figure D-5b shows the electric field lines and equipotentials due to two particles 1 and 2 with charges $+q$ and $-q$. Consider a point A located on the vertical line which is perpendicular to and bisects the line joining the particles 1 and 2. (a) What is the electric potential at A? (b) What is the electric work done on a particle of charge $+Q$ as it moves from A, along the vertical line to a point S very far from the particles 1 and 2? (c) Draw arrows indicating the directions of a displacement $\vec{D}$ of the particle along the path described, and of the electric force $\vec{F}$ on the particle due to the particles 1 and 2. Use this sketch to explain your answer to part (b). *(Answer: 24) (Suggestion: [s-16])*

Note: Tutorial section E contains further problems on relating descriptions of the electric interaction.

SECT.

F PROBLEMS

Relating Speed, Kinetic Energy, and Potential

The following problems require applying the relations of this unit together with conservation of energy and the definition $K = 1/2mv^2$ of kinetic energy (relations discussed in Chs. 14 and 15).

F-1 *Speeds produced by a Van-de-Graaff accelerator:* The charged sphere in a Van-de-Graaff accelerator produces at its surface a potential V. Consider a particle of mass m and charge q, which is initially at rest near the surface of this sphere, and which then accelerates until it moves with a final speed v when it is very far from the sphere. During this time the particle interacts appreciably only with the sphere. (a) Use the symbols provided to write expressions for the initial and final energies of this particle. (b) Apply conservation of energy to write an expression for the final speed v if the particle. (c) A proton has a mass of 1.7×10^{-27} kg and a charge of 1.6×10^{-19} C. What is the final speed of a proton accelerated by a sphere producing a potential of 3.4×10^5 volt at its surface? (d) An alpha particle (helium nucleus) has a charge twice that of the proton and a mass four times that of the proton. Is the final speed of an alpha particle accelerated by this sphere larger or smaller than the final speed of the proton? *(Answer: 23) (Suggestion: [s-13])*

F-2 *Measuring speed by using potential:* The speed of charged atomic particles can be measured by allowing these particles to enter the device shown in Fig. F-1 and adjusting the potential V so that the electric field between the two plates is just strong enough to slow the particles and bring them to rest just before they strike the detector. (a) Write expressions for the energy of a particle of mass m and charge q as it enters with speed v the region between the plates and as it comes to rest just before striking the detector. (b) Write an expression for the entering speed v of the particle. (c) If the magnitude of the potential V is 32 volt, and the particle is an electron with mass 1×10^{-30} kg, what is the initial speed v of the electron? *(Answer: 16)*

Assorted Applications of Potential

F-3 *Binding energy:* The "binding energy" of an electron in an atom is the energy which must be supplied to this electron by some external force in order to remove the electron from the atom and to place it at rest at some point far away. An electron in a hydrogen atom has a mass of 9×10^{-31} kg and moves with an average speed of 2×10^{6} m/s at an average of 0.5 Å from the proton which forms the atom's nucleus. Use these numbers to estimate the binding energy of the electron in a hydrogen atom. *(Answer: 26) (Suggestion: [s-11])*

F-4 *The potential due to a dipole:* Many charge distributions (including most of those in living tissue) arise when a particle of positive charge q is separated from a particle with negative charge $-q$. Fig. F-2 shows this common charge distribution, called a "dipole." Let us find the electric potential produced by a dipole at a point P some large distance R from the dipole's center. Notice that, because R is very large compared to the distance d between the particles, angles shaded in Fig. F-2 are all very nearly θ, and the angles marked by "$\neg$" are very nearly right angles. (a) Use the charge q and the distances r_1 and r_2 between P and particles 1 and 2 to express the potential V at P due to the dipole. (b) Notice that the distance x equals $(d/2)\cos\theta$ and use this fact to express the distances r_1 and r_2 in terms of R, d, and θ. (c) Express V in terms of q, r, d, and θ. Simplify your answer by writing it as a single fraction, and by noting that d^2 is negligibly small compared with R^2. (d) According to problem B-3 at a point like P' (where $\theta = 90°$) the potential V' produced by the dipole is zero. Is your answer to part (c) consistent with this earlier result? *(Answer: 20)*

F-5 *Potentials of biological systems:* The expression $V = k_e q \cos\theta / R^2$ can be used to describe very approximately the potentials produced by many biological systems. For example, the potential produced by the heart at a point P by a dipole which rotates (Fig. F-3a) so that R remains the same while θ changes with time t. The potential produced at a distant point by a depolarizing nerve cell is similar to the potential produced at a point P by a dipole moving along its axis (Fig. F-3b) so that as time t increases, R becomes first smaller then larger while θ changes from nearly 0° to 90° and then nearly to 180°. Which of the graphs shown in Fig. F-3c and F-3d shows the potential produced at P by the rotating dipole and which the potential produced by the dipole moving along its axis? *(Answer: 27) (Suggestion: [s-14])*

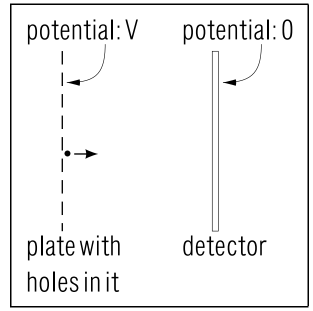


Fig. F-1.

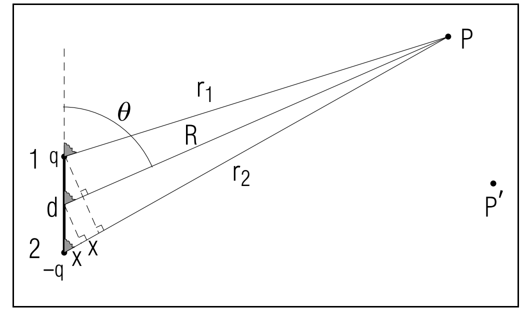


Fig. F-2.

Note: Tutorial section F contains further applications of potential.

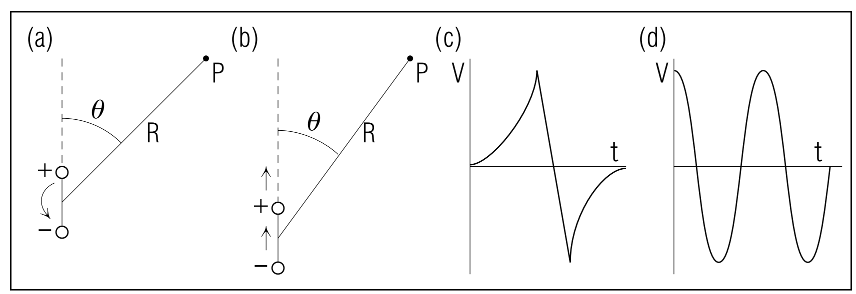


Fig. F-3.

TUTORIAL FOR A

REVIEW OF ELECTRIC FORCE, WORK, AND POTENTIAL ENERGY

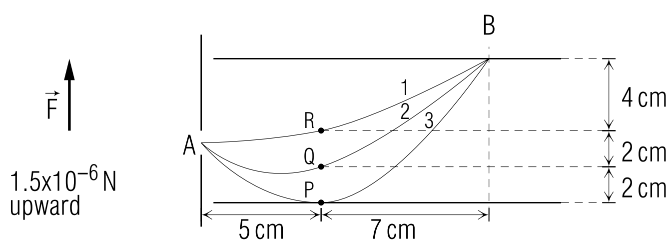
a-1 *PURPOSE:* Effective study of electric potential, the subject of this unit, depends on a thorough understanding of electric force, electric work, and electric potential energy. The following frames should help you to reestablish understanding of these quantities and of the relations between them. If you have difficulty with any of the following material, you should review the original discussion of these quantities in Chs. 14 and 15.

a-2 *WORK FOR CONSTANT FORCES AND SMALL DISPLACEMENTS:* Consider a particle acted on by a constant force $\vec{F}$ as it moves through a displacement $\vec{D}$ from an initial point A to a final point B. Then the work W_{AB} done on the particle by the force $\vec{F}$ is:

$$W_{AB} = F D_F$$

where D_F is the component of the displacement $\vec{D}$ along the direction of $\vec{F}$. [Recall that this relation applies to any force $\vec{F}$ for displacements $\vec{D}$ sufficiently small that (within desired precision) $\vec{F}$ remains constant for motion through $\vec{D}$.]

The following drawing shows two parallel charged plates. A particle enters the region between the plates through the hole at A, and depending on its initial velocity, travels along one of the three indicated paths to the point B. The particle interacts appreciably only with the plates, which exert on it a constant force of 1.5×10^{-6} N upward.



If the particle moves from A to B along path 1, what is the component D_F along $\vec{F}$ of the displacement from A to B?

► $D_F =$ ____________

Note: For help in finding components, see suggestion [s-12].

What is the work W_{AB} done by $\vec{F}$ on the alpha particle as it moves from A to B along path 1?

► $W_{AB} =$ ____________

What is the work done by $\vec{F}$ on the alpha particle if it moves from A to B along path 2 or along path 3?

► Along path 2, $W_{AB} =$ ____________

► Along path 3, $W_{AB} =$ ____________

Does the work done by a constant force on a particle depend on the particular path traveled by the particle, or just on the endpoints of this path?

► entire path, endpoints only

(Answer: 34) (Suggestion: [s-12].)

a-3 *PROPERTIES OF WORK:*

What is the single SI unit of work? Express this unit in terms of the SI units for force and distance.

► ____________ = ____________

In the diagram in the preceding frame, consider the paths from A to P, from A to Q, and from A to R.

For each of these paths, what is the component D_F along $\vec{F}$ of the displacement $\vec{D}$ from the path's beginning to its end? What is the corresponding value of the work done by $\vec{F}$ on a particle moving along this path?

►

	D_F	Work
A to P		
A to Q		
A to R		

Complete the following summary by using *positive*, *negative* or *zero*.

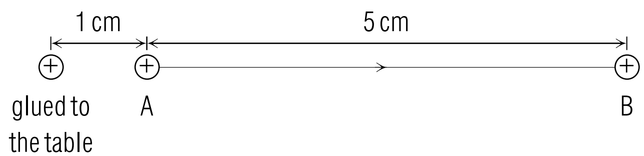
► If a displacement $\vec{D}$ is roughly along a force $\vec{F}$, then the corresponding work is ________ .

If $\vec{D}$ is roughly opposite to $\vec{F}$, then the work is ________ .

If $\vec{D}$ is perpendicular to $\vec{F}$, then the work is ________ .

(Answer: 38)

a-4 *APPLICABILITY OF $W_{AB} = FD_F$:* Consider two small balls, each with a positive charge of 2×10^{-8} C. One ball is glued to a smooth horizontal table top, and the other is placed 1 cm away so that it is free to move along the table top from A to B as indicated in this diagram.



What is the work W_{AB} done on the moving ball by the force due to the fixed ball?

(a) $W_{AB} = FD_F$ where $D_F = 5$ cm and $F = 0.36$ N is the magnitude of the force on the moving ball when it is at the point A.

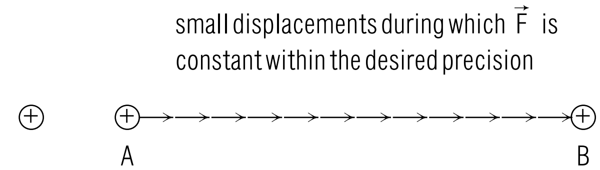
(b) $W_{AB} = FD_F$ where $D_F = 5$ cm and $F = 0.001$ N is the magnitude of the force on the moving ball when it is at the point B.

(c) Neither of these.

► (a), (b), (c).

(Answer: 31) (Suggestion: [s-2].)

a-5 *WORK DONE BY NON-CONSTANT FORCES:* To correctly find the work done by a non-constant force requires adding the small works done during displacements sufficiently small that the force is constant during each small displacement.



During each of the small displacements shown in the preceding diagram is the work done by the indicated force positive, or negative? Is the work W_{AB} positive or negative?

► ________ , ________

Suppose the fixed ball in the preceding diagram is *negatively* charged, so that the force $\vec{F}$ on the moving ball is directed toward the left.

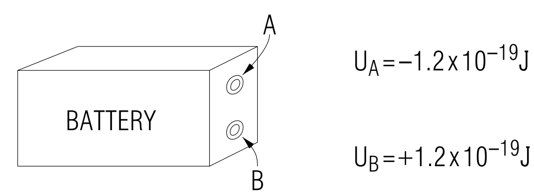
What then is the sign of the work W_{AB} done by $\vec{F}$ on the ball as it moves from A to B?

► ________

(Answer: 42)

a-6 *RELATING WORK TO POTENTIAL ENERGY:* Suppose a particle moves from a point A to a point B under the influence of interactions with other particles. If the moving particle's potential energy U due to these interactions is known, then it is very easy to find the corresponding work done on the particle. This work W_{AB} simply equals the drop $U_A - U_B$ in the particle's potential energy.

The following diagram shows a battery, and indicates the electric potential energy of an electron at each of the battery's terminals (A and B).



What is the electric work W_{BA} done on an electron as it moves from B to A through a portable radio?

► $W_{BA} =$ ____________________

What is the electric work W_{AB} done on an electron if it moves from A to B through the battery?

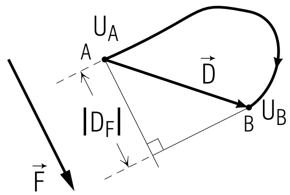
► $W_{AB} =$ ____________________

The total work done on an electron as it moves through a battery is the positive sum of the electrical and chemical work done on it.

(Answer: 39)

a-7 *SUMMARY:* The preceding frames should have enabled you to re-establish familiarity with the quantity work, and with its relation to force and to potential energy. The following table summarizes these relationships.

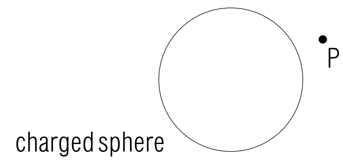
$U_A - U_B$	$=$	W_{AB}	$=$	$F D_F$
for interaction described by potential energy U				for a force constant during the displacement $\vec{D}$



Now: Go to the beginning of text section A.

Understanding the Definition of Electric Potential (Cap. 1a)

a-8 *STATEMENT AND EXAMPLE:* A small plastic ball with charge q is suspended at a point P near a charged metal sphere. As a result of the electric interaction between these two objects, the plastic ball has a potential energy U.



(a) Express the potential V produced by the metal sphere at the point P in terms of the symbols provided.

► $V =$ ____________________

(b) If $q = 2 \times 10^{-8}$ C and $U = 6 \times 10^{-5}$ J, what is the potential at P?

► $V =$ ____________________

(c) If a small plastic ball of charge -2×10^{-8} C is suspended at P, then its electric potential energy is just the opposite of the value given in part (b), or $U = -6 \times 10^{-5}$ J. Use these new values to find the potential at P due to the metal sphere.

► $V =$ ____________________

(d) Are your two values for the potential at P consistent?

► yes, no

(Answer: 49)

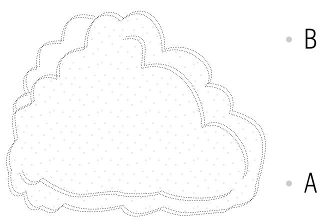
a-9 *PROPERTIES:* Use the following table to summarize the properties of the quantities indicated:

►

	Electric potential	Electric potential energy	Electric field	Electric force
algebraic symbol				
simplest SI unit				
number or vector				
possible signs				

(Answer: 47)

a-10 *DEPENDENCE:* The following diagram shows two points, A and B, near a charged cloud.



Two raindrops, having different charges q and q', are successively at the point A. When located at A, these particles have electric potential energies U and U' due to the cloud. The potential produced at A by the cloud can be found either by dividing U by q or by dividing U' by q'.

(a) Are these two values for the potential at A equal?

► equal, not equal, might be equal or not equal

(b) Is the potential V_B produced at B by the cloud equal to the potential V_A produced at A by the cloud?

► equal, not equal, might be equal or not equal

Suppose another charged cloud moves nearby.

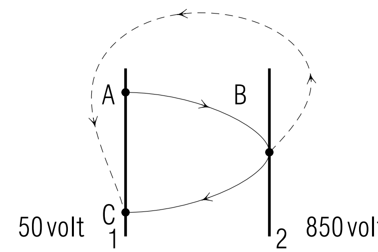
(c) Is the new potential at A (due to the two clouds) equal to the original potential at A?

► equal, not equal, might be equal or not equal

(Answer: 53)

Relating Work to Potential Drop (Cap. 3)

a-11 *FINDING POTENTIAL DROP:* The two uniformly charged parallel plates shown in the following diagram produce a uniform electric field in the region between them. All points on plate 1 have a potential of 50 volt and all points on plate 2 have a potential of 850 volt.



A proton moves between these plates along the path from A to C indicated by the solid line in the preceding diagram.

What are the potential drops along the paths from A to B, from B to C, and from A to C?

► $V_A - V_B =$ ____________

$V_B - V_C =$ ____________

$V_A - V_C =$ ____________

What is the potential drop from B to C along the dotted path shown in the diagram?

$V_B - V_C =$ ____________

(Answer: 50) (Suggestion: [s-3].)

a-12 *FINDING WORK AND DROP IN POTENTIAL ENERGY:* Because $V = U/q$, the drop in a particle's potential energy as it moves from a point A to a point B is given by:

$$U_A - U_B = q(V_A - V_B)$$

Consider the proton of charge 1.6×10^{-19} C described in the preceding frame. This proton interacts appreciably only with the two charged plates.

Use the values for potential drop found in the preceding frame to find the drop in the proton's potential energy and the work done on the proton as it moves from A to B, from B to C, and from A to C.

► $U_A - U_B =$ ____________ , $W_{AB} =$ ____________

$U_B - U_C =$ ____________ , $W_{BC} =$ ____________

$U_A - U_C =$ ____________ , $W_{AC} =$ ____________

(Answer: 56) Now: Go to text problem A-1.

TUTORIAL FOR B

POTENTIAL ENERGY Of ONE PARTICLE DUE TO ANOTHER

In order to discuss the electric potential produced by one particle, it is essential first to re-establish familiarity with the potential *energy* of one particle due to another. The following frame should help you to review this quantity.

b-1 *THE DEFINITION OF POTENTIAL ENERGY:* Consider a charged particle X which is located at a point A and which is acted on by electric forces due to other nearby particles. The electric potential energy U_A of the particle X is defined as the work W_{AS} done by electric forces as X moves from A to some standard position S (usually very far from all other particles).

$$U_A = W_{AS}$$

b-2 *THE SIGN OF POTENTIAL ENERGY:* Consider a charged particle 2 at a point A near a fixed charged particle 1.

If both 1 and 2 are positively charged, is the force on 2 due to 1 attractive or repulsive? Is the force on 2 directed towards the right or towards the left?

► attractive, repulsive

right, left

To find the sign of the potential energy U_A of 2 due to its interaction with 1, suppose that 2 were to move from its current position at A to some standard position S very far away.

Is the work W_{AS} done on particle 2 by particle 1 positive or negative? Is the potential energy U_A of 2 at A positive or negative?

► W_{AS}: positive, negative

U_A: positive, negative

Suppose the fixed particle 1 is now negative, while the particle 2 is still positive.

Is the force on 2 due to 1 now attractive or repulsive?

► attractive, repulsive

What now is the sign of the potential energy U_A of 2 at A?

► positive, negative

(Answer: 35)

b-3 *THE MAGNITUDE OF POTENTIAL ENERGY:* Suppose that the particles 1 and 2 described in the preceding frame have charges q_1 and q_2, and that the point A is a distance R from particle 1.

Which of the following is the correct expression for the magnitude of U_A, the potential energy of particle 2 at the point A?

(a) $k_e q_1 q_2/R^2$

(b) $k_e q_1/R^2$

(c) $k_e q_1 q_2/R$

(d) $k_e q_1/R$

► (a), (b), (c), (d)

(Answer: 44) (Suggestion: [s-15].)

b-4 *SUMMARY:* The preceding frames should have helped you to recall this important characteristics of the potential energy U of one charged particle due to its interaction with another:

This potential energy is positive or negative according to whether the two particles have charges with the same or opposite signs [The reason is that a repulsive force does positive work $W_{AS} = U_A$, while an attractive force does negative work W_{AS}.]

The magnitude of this potential energy U is:

$$U = k_e q_1 q_2/R$$

where R is the separation of the two particles, and q_1 and q_2 are their charges.

If the particle of charge q_2 is located at some point S very far from particle 1, then its potential energy U_S is just

$$U_S = 0$$

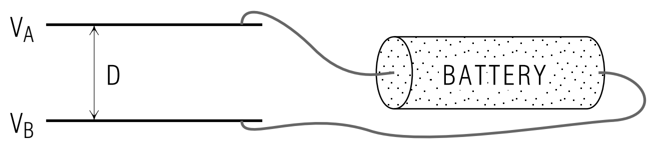
The preceding frames have certainly *not* provided a full explanation of these results. If you are confused, or wish further explanation, you should review Unit 415, especially Sec. D.

Now: Go to the beginning of text section B.

TUTORIAL FOR E

FURTHER PROBLEMS ON RELATING DESCRIPTIONS OF ELECTRIC INTERACTIONS (Cap. 3)

e-1 *ELECTRIC FIELD AND POTENTIAL BETWEEN PARALLEL PLATES:* When the terminals of a battery are connected to two parallel metal plates, all points on a plate have the potential of the battery terminal to which the plate is attached. Each plate is uniformly charged, and so the electric field $\vec{E}$ between the plates is uniform. Thus if two plates A and B are connected to a 9 volt battery, the potential difference $|V_A - V_B|$ is 9 volt.



(a) Express the magnitude E of the electric field between two parallel metal plates in terms of the potential difference $|V_A - V_B|$ between the plates, and the distance D separating the plates. (b) To produce a large field between two parallel metal plates, should the potential difference between the plates be large or small? Should the separation between the plates be large or small? (c) What is the magnitude of the electric field between two plates connected to a 9 volt battery and separated by 0.3 cm? (d) The opposite edges of a sheet of nerve membrane become uniformly charged through a chemical process similar to those occurring in a battery. If the potential drop across membrane of thickness 50 Å (5×10^{-9} m) is 0.1 volt, what is the magnitude of the electric field within this membrane? *(Answer: 55)*

e-2 *PRODUCING A LARGE POTENTIAL NEAR A VAN-DE-GRAAFF SPHERE:* The charged sphere in a Van-de-Graaff accelerator should produce at its surface as large a potential as possible (so that the work done on charged particles moving near this surface will be large). The largest magnitude $|Q|$ of charge which can be placed on such a sphere in air is that Q which produces at the surface of the sphere an electric field equal in magnitude to the breakdown strength of air. (a) Express this maximum magnitude $|Q|$ of charge in terms of the radius R of the sphere and the breakdown strength E. (b) Use the preceding expression

for $|Q|$ in terms of E and R to express the maximum possible magnitude $|V|$ of the electric potential at the surface of a sphere. (c) To produce a potential of large magnitude near the surface of a sphere, should the radius of this sphere be large or small? *(Answer: 45)*

Now: Go to text section F.

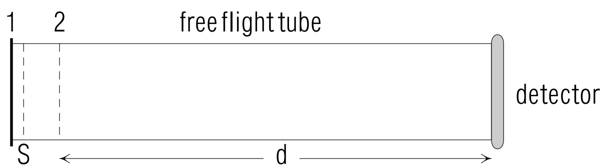
TUTORIAL FOR F

ADDITIONAL PROBLEMS

The following problems concern applications of potential to finding the charge of the earth ([f-1]), to using a mass spectrometer for identifying charged atomic particles ([f-2]), and to measuring and describing the electric activity of nerve and muscle tissue ([f-3], [f-4]).

f-1 *THE CHARGE AND POTENTIAL OF THE EARTH:* The earth produces at its surface an electric field which is directed upward and which has a magnitude of 100 V/m. For the precision desired here, the earth is a sphere of radius 6.4×10^6 meter, and its charge Q is distributed uniformly over its surface. What is the charge of the earth? What is the potential at a point on the earth's surface (relative to a standard position very far away)? *(Answer: 57)*

f-2 *MEASURING PARTICLE MASS WITH A MASS SPECTROMETER:* The following diagram indicates the essential parts of a time-of-flight mass spectrometer, a device for measuring the masses of particles which all have charge $-e$ because they have been bombarded with electrons.



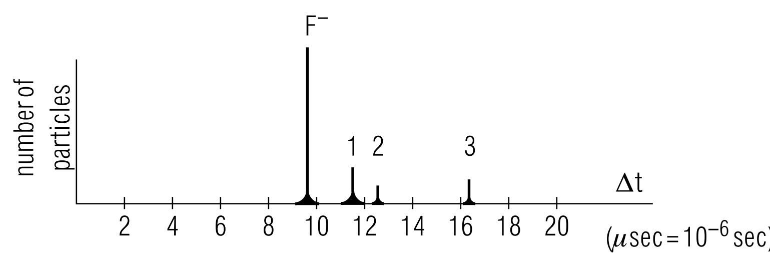
The plate 1 and screen 2 have uniformly distributed charges of equal magnitude but opposite sign. They thus produce a uniform electric field in the region between them, but produce a negligible field in the region outside. The plate and screen each have uniform potentials which differ by $V_2 - V_1$.

The negatively charged particles are initially confined near the plate by the repulsive electric forces due to a second screen (indicated by S on the diagram) which is negatively charged. When the screen S is suddenly discharged, the particles are free to move under the sole influence of the plate 1 and screen 2, and are thus accelerated towards the right until they pass through the holes in screen 2 and into the free-flight tube.

The particles then move freely until they reach a detector located a distance d from screen 2. A timer measures the "time of flight" Δt between the instant at which the particles pass through the screen 2 and the instant at which particles arrive at the detector. [The entire apparatus is enclosed in an evacuated container so that the particles interact appreciably only with the metal plates and screens.] We can in the following way relate the unknown mass m of a moving particle to the measurable potentials V_1 and V_2 to the measurable distance d and time interval Δt, and to the known magnitude e of the particle's charge.

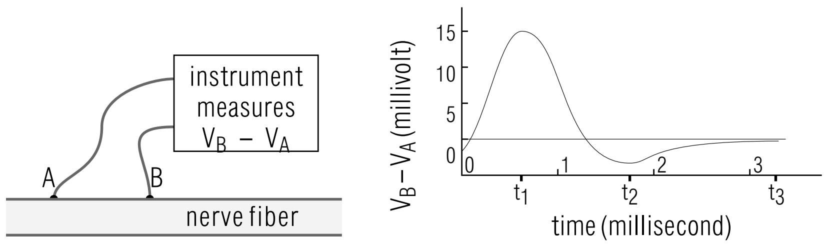
(a) Use the difference $V_2 - V_1$ in potentials to express the kinetic energy of a particle of mass m and charge $-e$ as it emerges through the holes in screen 2. (b) Express this energy in terms of the distance d and time interval Δt. (c) Write an expression for the mass m of the particle in terms of the known and measurable quantities described in this problem.

The following diagram shows the number of particles arriving at the detector as a function of the time of flight Δt. The charged particles described by this graph are all fragments of the compound Hexafluorethane, C_2F_6.



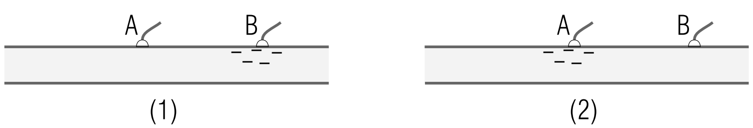
(d) Is the time flight of a $F_2{}^-$ ion $\sqrt{2}$, 2, or 4 times as long as the time of flight of a F^- ion which has a mass half as large? (e) The "peak" indicating the arrival of F^- ions is indicated on the preceding diagram. Which "peak" indicates the arrival of the $F_2{}^-$ ions? *(Answer: 54)*

f-3 *THE "ACTION" POTENTIAL OF A NERVE FIBER:* A simple means of studying the electric activity of nerve fibers involves connecting two points on the nerve surface (A and B in the following diagram) to an instrument which measures the potential difference $V_B - V_A$.



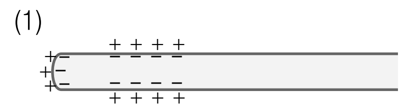
Commonly this instrument draws a graph (like the one shown at the right in the preceding diagram) of the potential difference $V_B - V_A$ as a function of time t. The characteristic pattern of changing potential differences as an electric impulse travels along the nerve is called the "action potential" of the nerve.

(a) At each of the times t_1, t_2, t_3, t_4 indicated on the graph, is the potential V_B larger than, smaller than, or equal to the potential V_A? (b) At each of these three times, is the electric field along the nerve surface between A and B directed roughly towards the right, or towards the left. (c) These observations can be explained by saying that a region of negative charge propagates along the nerve surface from left to right. Which of the following diagrams best describes the charge distributions on the nerve surface at the time t_1 and at the time t_2? *(Answer: 46)*

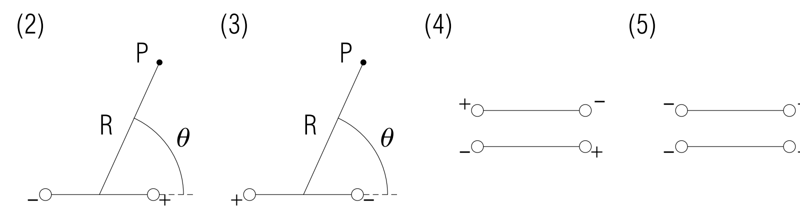


f-4 *ELECTRIC POTENTIAL DUE TO NERVE AND MUSCLE TISSUE:* (The following discussion depends on the results of text problem F-4.) When a muscle or nerve cell is inactive, the surfaces of its membrane are uniformly charged, positively on the outside and negatively on the inside. When the cell becomes active (by contracting or conducting a nerve impulse), charged ions migrate across the membrane causing these surfaces to become neutral. This process begins at one end of the cell and occurs successively down its length so that at one instant during this process the charge distribution of a nerve or muscle cell looks something like that shown in the following diagram. This charge distribution can be described as a collection of small dipoles aligned perpendicular to the surface of the cell. To estimate the potential

produced by such a cell, let us see how to estimate the potential produced at some distant point by a collection of dipoles.

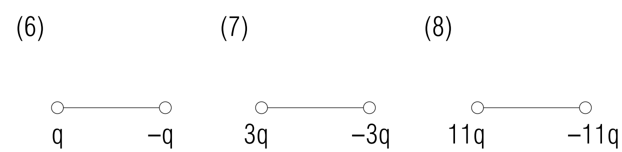


The following diagram shows several collections of dipoles, each consisting of two particles of charge q and $-q$ which are separated by a distance d. (a) Write an expression for the magnitude of the potential at P due to the dipole in diagram (2). What is the sign of this potential? [Notice that P is closer to the positively charged particle than to the negatively charged one.] What is the sign of the potential at P due to the dipole shown in diagram (3)?



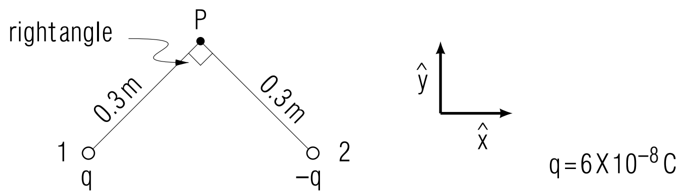
(b) The preceding diagrams (4) and (5) show two pairs of dipoles. Which pair produces at any distant point P a potential approximately equal to twice the potential produced by a single dipole? Which pair produces at P a potential which is very small compared with the potential due to one dipole?

(c) The preceding observations suggest that in estimating the potential produced at a distant point P by a collection of dipoles, one can neglect the effect of pairs of oppositely oriented dipoles, and simply add the potentials produced by the remaining aligned dipoles. If each + and − sign in diagram (1) indicates a charge of magnitude q, and separation d, which of the following diagrams shows a dipole which produces at a distant point P approximately the same potential as the nerve cells shown in diagram (1)? *(Answer: 51) (Suggestion: [s-5])*



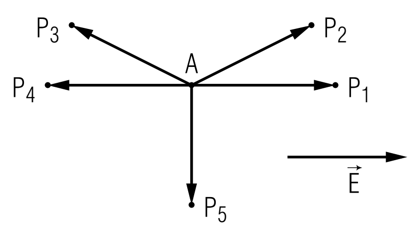
PRACTICE PROBLEMS

p-1 *DESCRIBING POTENTIAL DUE TO ANY NUMBER OF PARTICLES (CAP. 2):* At the point P shown in the following diagram, what are the electric field and the electric potential due to particle 1, due to particle 2, and due to both particles? The answer will contain q. *(Answer: 40) (Suggestion: Review text problems B-2 and B-3.)*



p-2 *THE POTENTIAL DUE TO ANY NUMBER OF PARTICLES (CAP. 2):* What is the charge Q of a sphere of radius 30 cm which produces at its surface a potential of 3000 volt? *(Answer: 32) (Suggestion: Review text problem B-5.)*

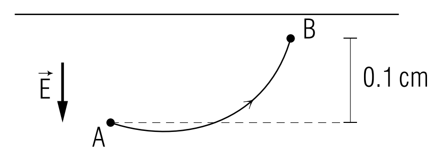
p-3 *RELATION BETWEEN POTENTIAL AND FIELD (CAP. 1C):* The following diagram shows a point A in a region in which the electric field $\vec{E}$ is uniform, and various possible displacements $\vec{D}$ all of magnitude $|\vec{D}|$ and beginning at the point A. These displacements end at the points P_1 through P_5.



(a) For each of the five displacements $\vec{D}$, is the corresponding component D_E positive, negative, or zero? Is the corresponding potential drop along $\vec{D}$ positive, negative, or zero? (b) At each of the points P_1 through P_5, is the potential larger than at A, smaller than at A, or the same as at A? (c) Complete the following statements by using *increases, decreases*, or *remains the same*. Along a displacement in roughly the direction of $\vec{E}$, the potential V ______________ . Along a displacement roughly opposite to $\vec{E}$, the potential V ______________ . Along a displacement exactly perpendicular to $\vec{E}$, the potential V ______________ . *(Answer: 36) (Suggestion: Review text section C, and text problems C-1 and C-2.)*

p-4 *RELATION BETWEEN POTENTIAL AND FIELD (CAP. 1C):* (a) What is the single SI unit of potential drop? (b) The electric field can be expressed in terms of the unit N/C, which corresponds to the definition $\vec{E} = \vec{F}/q$ of electric field. Use the definition of volt to express the unit of electric field in terms of volt. (c) Check that the unit of ED_E is properly equal to volt, since $ED_E = V_A - V_B$. *(Answer: 43) (Suggestion: Review text section C and text problems C-1 and C-2.)*

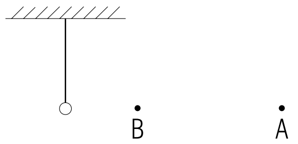
p-5 *RELATING QUANTITIES DESCRIBING ELECTRIC INTERACTION (CAP. 3):* The following diagram shows two uniformly charged parallel plates separated by 0.25 cm. Between these plates the electric field has a magnitude equal to 2.0×10^6 V/M, (two thirds the breakdown strength of air).



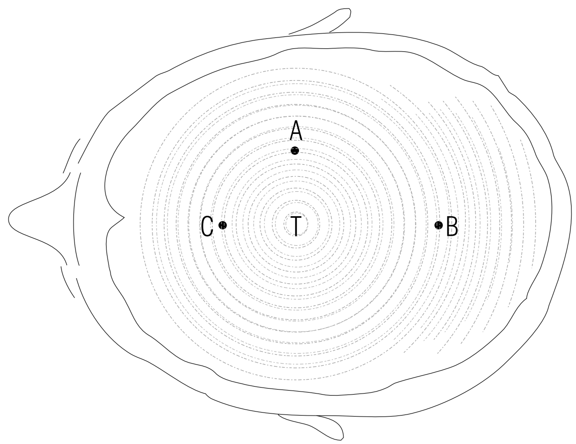
(a) What is the potential drop from the point A to the point B shown in the diagram? (b) What is the electric work done by the charged plates on an ion of charge -1.6×10^{-19} C moving along the indicated path from A to B? *(Answer: 41) (Suggestion: Review text problems C-3, C-4, and C-5.)*

p-6 *RELATING QUANTITIES DESCRIBING ELECTRIC INTERACTION (CAP. 3):* A small plastic ball has a uniformly distributed charge of -2.0×10^{-8} C. (a) What is the potential drop from a point A located 3 cm from the center of this ball to a point B located 1 cm from its center? (b) What is the electric work done by this ball on a metal particle of charge 1.0×10^{-9} C as the particle moves from A to B?

(Answer: 37) (Suggestion: Review text problems C-3, C-4, and C-5.)



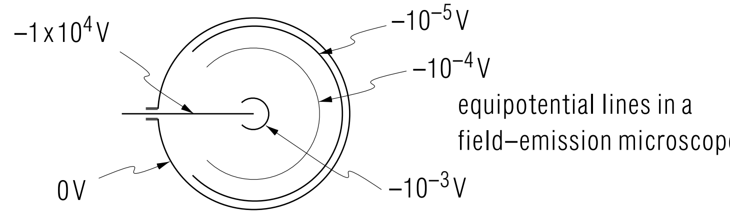
p-7 *USING EQUIPOTENTIALS TO DESCRIBE FORCES AND FIELDS (CAP. 4A):* Measurements of electric potential due to electric activity of the brain can be used in diagnosing tumors and neural disfunction. The following diagram shows approximately the normal equipotential lines on the skull surface at one instant during sleep. Lines near the top of the head correspond to larger values of potential.



Describe the direction of the electric field at the points A, B, and C. *(Answer: 33) (Suggestion: Review text problems D-1 and D-2.)*

p-8 *USING EQUIPOTENTIALS TO FIND WORK (CAP. 4B):* In field emission microscopy, a large potential difference is produced between the surface of a glass sphere with a thin metal coating and the tip of a fine wire which is located at the center of the sphere. The electric field near the tip of the wire is then sufficiently large that electrons are pulled from it and travel along approximately radial paths until they strike a phosphorescent coating on the glass sphere. The sphere then glows where it is struck by the electrons and so

provides a picture of the arrangement of the atoms and molecules on the wire tip. (a) Use the following diagram of equipotential lines to find the work done on an electron as it moves from a point on the wire to a point on the glass sphere. Express your answer in terms of eV and J. *(Answer: 48) (Suggestion: Review text problems D-3 and D-4.)*



SUGGESTIONS

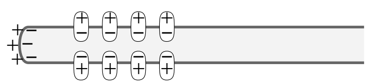
$\boxed{\textbf{s-1}}$ *(Text problems D-3 and D-4):* Recall from Table C-1 that $W_{AB}/q = V_A - V_B$.

$\boxed{\textbf{s-2}}$ *(Tutorial frame [a-4]):* The relation $W_{AB} = FD_F$ directly applies only to a displacement $\vec{D}$ which is small enough that the force $\vec{F}$ has one constant value as the particle moves through this displacement.

$\boxed{\textbf{s-3}}$ *(Tutorial frame [a-11]):* All points on plate 1 (including A and C) have a potential of 50 volt. All points on plate 2 (including B) have a potential of 850 volt. Use these values to find the potential drop (the initial potential minus the final potential) for the indicated paths.

$\boxed{\textbf{s-4}}$ *(Text problem B-1):* Distinguish carefully between the expression for the *potential* due to one particle (which involves only the charge of that one particle) and the expression for the *potential energy* of one particle due to a second particle (which involves the charges of both particles).

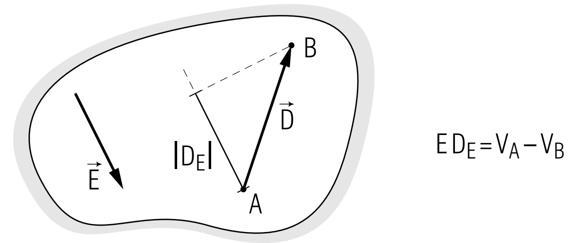
$\boxed{\textbf{s-5}}$ *(Tutorial frame [f-4]):* At a very distant point P a negligible potential is produced by the pairs of oppositely oriented dipoles indicated in the following diagram. Thus the potential at P is almost entirely produced by the three aligned dipoles at the left.



$\boxed{\textbf{s-6}}$ *(Text problem C-1):* (b) Notice that the potential drop from an initial point P to a final point Q is the initial potential V_P minus the final potential V_Q. Thus if a potential drop is positive, the initial potential is *larger* than the final potential, and the potential decreases from P to Q. Further, because for *any* points A and B, $V_A - V_B = ED_E$, the potential always *decreases* along a displacement $\vec{D}$ which is directed roughly *along* $\vec{E}$ (so that D_E is positive).

(c) Recall the definition: volt = J/C = N m/C.

$\boxed{\textbf{s-7}}$ *(Text problem E-3):* The relation $ED_E = V_A - V_B$ must be true for any displacement $\vec{D}$ between points A and B in the region considered.



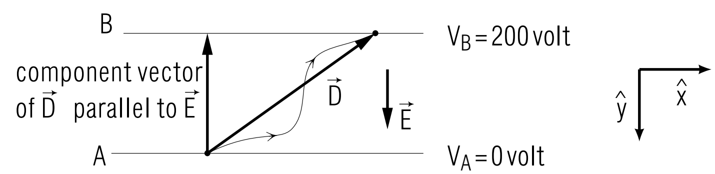
(a) If $\vec{E} = 0$, then $V_A = V_B$ for any two points in the region.

(b) If $V_A = V_B = 0$, then $ED_E = 0$ for any displacement $\vec{D}$, and so $\vec{E} = 0$ throughout the region.

$\boxed{\textbf{s-8}}$ *(Text problem C-5):* (a) To find the potentials at the points A and B, you will need the relation (discussed in Sec. B) which describes the potential due to a spherically symmetric charge distribution.

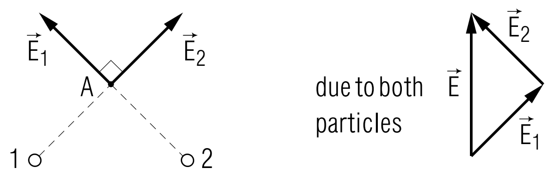
(c) Recall that positive work on a particle is done by a force which is roughly along the particle's path and which therefore tends to increase the particle's speed. Negative work on a particle is done by a force which is roughly opposite to the particle's path and which therefore tends to decrease the particle's speed.

$\boxed{\textbf{s-9}}$ *(Text problem C-3):* Consider any path from a point on plate A to a point on plate B.



Then use the relations summarized in Table C-1 to find the desired quantities. Be careful of signs in finding the work W_{AB} in part (c) since $\vec{E}$ and the component vector of $\vec{D}$ along $\vec{E}$ have opposite signs.

$\boxed{\textbf{s-10}}$ *(Text problem B-3):* To find the electric field at A or B, use arrow symbols to add the electric fields $\vec{E}_1$ and $\vec{E}_2$ due to each of the two particles. For the point A, your diagram might look like this:

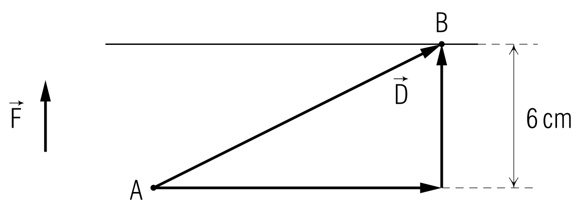


Notice that the $\vec{E}_1$ and $\vec{E}_2$ are perpendicular and that they have the same magnitude $k_e q/(0.060\,\text{m})^2$.

s-11 *(Text problem F-3):* First find the energy of the electron in its initial state within the atom. This energy is the sum of the electron's positive kinetic energy and negative potential energy (due to its interaction with the proton). In the final state, the electron is at rest and far from the proton, and so has an energy of zero. The binding energy is the energy which must be supplied to enable the electron to go from its initial state in the atom to this final state.

The electron and proton each have a charge of magnitude 1.6×10^{-19} C. The mass of the electron is 1×10^{-30} kg. $1\,\text{Å} = 10^{-10}$ meter.

s-12 *(Tutorial frame [a-2]):* As indicated in the following diagram, a particle moving along any path from A to B moves through a displacement $\vec{D}$ which has a component vector along $\vec{F}$ of magnitude 6 cm.

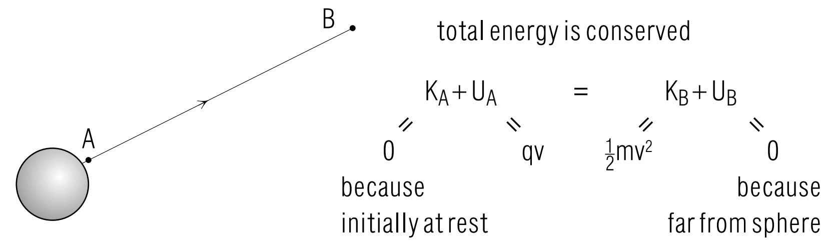


Therefore the work done by $\vec{F}$ on a particle moving from A to B is positive and has a magnitude of FD_F or $F(0.06\,\text{m})$.

(Note: If you need further help with finding and using component vectors, review the discussion in text section B of Unit 407 and its associated tutorial.)

s-13 *(Text problem F-1):* The following diagram shows a point A where the particle is initially located and a point B where the particle

is finally located. The diagram at the right shows the relations between various quantities describing the electric interaction of the particle and the Van-de-Graaff sphere.



The symbols K_A, K_B, U_A, and U_B describe the kinetic and electric potential energies of the particle at the points A and B.

s-14 *(Text problem F-5):* Since R is constant for the rotating dipole, the corresponding potential is proportional to $\cos\theta$. Thus as θ increases with time t, the graph of V should look like the graph of a cos (or sin) function. Notice that the moving dipole produces first a positive potential, and then later (after passing P) a negative potential. When the dipole is far from the point P, the potential it produces must also be small because this potential is proportional to $1/R^2$.

s-15 *(Tutorial frame [b-3]):* Let us review the meaning of each of the four expressions in tutorial frame [b-3]:

(a) $k_e|q_1q_2|/R^2$ is the magnitude of the electric force on one of the two particles due to the other.

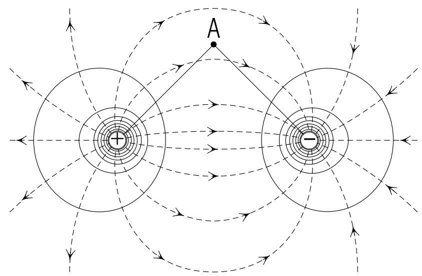
(b) $k_e|q_1|/R^2$ is the magnitude of the electric field at the point A due to particle 1.

(c) $k_eq_1q_2/R$ is the magnitude of the electric *potential energy* of the interaction of particles 1 and 2.

(d) As we shall discuss in text section B, k_eq_1/R is the magnitude of the electric *potential* at the point A due to particle 1.

(Note: For a more complete discussion of electric force and potential energy, see text section D of Unit 415.)

s-16 *(Text problem E-4):* The following diagram shows text Fig. D-5b and a point A which is a distance d from each of the particles 1 and 2.



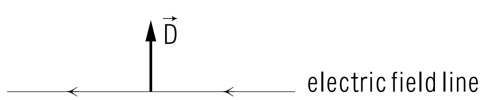
(a) Write expressions (in terms of q and d) for the electric potentials at A due to particle 1, due to particle 2 and due to both particles.

(b) Recall that at a point S very far from a collection of charged particles, the potential $V_S = 0$. If necessary, review Table C-1 in order to relate work to the potential drop $V_A - V_S$.

(c) The following diagram shows a displacement $\vec{D}$ along the path from A to S.

From the beginning point of $\vec{D}$, draw an arrow indicating the direction of the electric force on the particle at this point.

►



What is the component D_F of $\vec{D}$ parallel to $\vec{F}$? What is the work $W = FD_F$ done by the electric force on the particle as it moves through the displacement $\vec{D}$?

► $D_F =$ ________________

► $W =$ ________________

(Answer: 52)

ANSWERS TO PROBLEMS

1. a. $V_B - V_A = -1.5\,\text{volt}$

   b. Along either path, $W_{BA} = 1.5\,\text{eV} = 2.4 \times 10^{-19}\,\text{J}$

2. $\vec{E} = 0$, $V = 4k_eq(1/\sqrt{2}) = 4\sqrt{2}k_eq/L$

3. a. $4 \times 10^4\,\text{V/m}$ directed from B to A, and perpendicular to plates.

   b. $6 \times 10^{-15}\,\text{N}$

   c. $-3 \times 10^{-17}\,\text{J}$

4. a. $-3 \times 10^{-10}\,\text{J}$

   b. yes, different, $9 \times 10^{-10}\,\text{J}$; no, the same

5. a. $k_eq_1/R = 6.0 \times 10^3\,\text{volt}$

   b. $-1.8 \times 10^{-4}\,\text{J}$, $-9.0 \times 10^3\,\text{volt}$, $-1.8 \times 10^{-4}\,\text{J}$

6. a. $2.0 \times 10^5\,\text{volt}$

   b. $6.4 \times 10^{-14}\,\text{J}$

   c. increase

7. a. $1 \times 10^6\,\text{eV}$

   b. 0

   c. $1 \times 10^6\,\text{eV}$

   d. $2 \times 10^6\,\text{eV}$

   e. $2 \times 10^6\,\text{eV}$

8. a. $V_P - V_Q = ED_E$

   b. decreases

   c. $3 \times 10^2\,\text{volt}$

9. a. $V_B = k_eq_1/R$, $E_B = k_eq_1/R^2$, $U_B = k_eq_1q_2/R$, $F_{BA} = k_eq_1q_2/R^2$

   b. V_B, volt; E_B, N/C; U_B, J; F_{BA}, N

10. a. $-200\,\text{volt}$

    b. $-3.2 \times 10^{-17}\,\text{J}$

    c. $-3.2 \times 10^{-17}\,\text{J}$, yes

11. a. $V_A = 2.7 \times 10^5\,\text{volt}$, $U_A = 8.6 \times 10^{-14}\,\text{J}$

    b. $W_{AS} = U_A = 8.6 \times 10^{-14}\,\text{J}$

12. At A, $\vec{E} = 7.1 \times 10^4\,\mathrm{N/C}\ \hat{y}$, $V = 6.0 \times 10^3$ volt. At B, $\vec{E} = 0$, $V = 8.6 \times 10^3$ volt. Except at special points like B, the number V is easier to find than the vector $\vec{E}$.
13. a. -20 volt
    b. E changes value as particle moves through $\vec{D}$. Thus there is no one value for E to use in the expression ED_E.
14. a. 2.0×10^2 N/C directed towards the left
    b. 3.2×10^{-17} N directed towards the left
15. a. right, central wire
    b. $1750\,\mathrm{eV} = 2.8 \times 10^{-16}$ J
16. a. $(1/2)mv^2 + qV$, 0
    b. $V = \sqrt{-2qV/m}$
    c. 3×10^6 m/s
17. $400\,\mathrm{eV} = 6.4 \times 10^{-17}$ J
18. a. no, yes
    b. yes, yes
19. a. 2×10^{-2} N/C directed upwards and towards the left
    b. 3×10^{-21} N directed downwards and towards the right
20. a. $(k_e q/r_1) + (-k_e q/r_2)$
    b. $r_1 = R - (d/2)\cos\theta$, $r_2 = R + (d/2)\cos\theta$
    c. $V = k_e q/(R - (d/2)\cos\theta) + (-k_e q)/(R + (d/2)\cos\theta) = k_e q(d\cos\theta)/R^2$
    d. yes, because $\cos 90° = 0$
21. a. 30 volt (one significant figure)
    b. $-30\,\mathrm{eV}$ = -(5 or 6) $\times 10^{-18}$ J
22. -3×10^{-7} J (Note sign)
23. a. initial, qV; final, $(1/2)mv^2$
    b. $v = \sqrt{2qV/m}$
    c. 8.0×10^6 m/s
    d. smaller
24. a. 0

    b. 0
    c. $\vec{D}$ and $\vec{F}$ are perpendicular. Thus the work $W = FD_F$ done on the particle by the force $\vec{F}$ is zero.
25. (c)
26. (2 or 3) $\times 10^{-18}\,\mathrm{J} = 12$ to 19 eV
27. rotating, (d); moving along axis, (c)
31. (c)
32. 1.0×10^{-7} C
33. At all three points, the electric field is directed away from the point T at the top of the head.
34. $D_F = 6\,\mathrm{cm} = 0.06\,\mathrm{m}$; $W_{AB} = 9 \times 10^{-8}$ J for all paths from A to B. [Recall J = joule = N × m.]; end points only
35. repulsive, right; positive, positive; attractive, negative
36. a. both D_E and potential drop are positive for 1 and 2, negative for 3 and 4, and zero for 5.
    b. smaller at 1 and 2, larger at 3 and 4, the same at 5.
    c. decreases, increases, remains the same
37. a. 1.2×10^4 volt
    b. 1.2×10^{-5} J
38. joule = newton meter; A to P, $D_F = -0.02\,\mathrm{m}$, $W_{AP} = -3 \times 10^{-8}$ J; A to Q, $D_F = 0$, $W_{AQ} = 0$; A to R, $D_F = 0.02\,\mathrm{m}$, $W_{AR} = 3 \times 10^{-8}$ J; positive, negative, zero
39. $W_{BA} = +2.4 \times 10^{-19}$ J, $W_{AB} = -2.4 \times 10^{-19}$ J
40. particle 1, $\vec{E} = 6 \times 10^3$ N/C directed upward and towards the right. $V = 2 \times 10^3$ volt; particle 2 $\vec{E} = 6 \times 10^3$ N/C directed downwards and to the right, $V = -2 \times 10^3$ volt, both particles, $\vec{E} = 8 \times 10^3\,\mathrm{N/C}\ \hat{x}$, $V = 0$
41. a. -2.0×10^3 volt
    b. 3.2×10^{-16} J
42. positive, positive, negative

43. a. volt

b. $N/C = V/m$

c. ED_E has the unit $(V/m)m$ = volt

44. (c)

45. a. $|Q| = ER^2/k_e$

b. $|V| = RE$

c. large

46. a. t_1, larger; t_2, smaller; t_3, equal

b. t_1, left; t_2, right; t_3, zero

c. t_1, 2; t_2, 1

47.

	Potential	Potential energy	Electric Field	Electric Force
alg'c symb.	V	U	$\vec{E}$	$\vec{F}$
std. SI unit	volt	joule	N/C	N
number/vector	number	number	vector	vector
possible signs	$+, -, 0$	$+, -, 0$	not applicable to vectors	

48. a. 1×10^4 eV$= 2 \times 10^{-15}$ J

49. a. $V = U/q$

b. $V = 3 \times 10^3$ volt

c. $V = 3 \times 10^3$ volt

d. Two values should be the same.

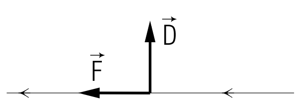
50. $V_A - V_B = -800$ volt, $V_B - V_C = 800$ volt, $V_A - V_C = 0$ volt $V_B - V_C =$ 800 volt (independent of path from B to C)

51. a. $2k_e qd \cos\theta/R^2$, $+$, $-$

b. 5,4

c. 7

52.



$D_F = 0$, $W = 0$

53. for (a), equal; for (b) and (c), might be equal or not equal

54. a. $e(V_2 - V_1)$

b. $(1/2)m(d/\Delta t)^2$

c. $m = 2e(V_2 - V_1)(\Delta t)^2/d^2$

d. $\sqrt{2}$

e. 2

55. $E = |V_A - V_B|/D$

b. large potential difference, small separation

c. 3×10^3 V/m

d. 2×10^7 V/m

56. $U_A - U_B = W_{AB} = -800$ eV$= -1.3 \times 10^{-16}$ J,

$U_B - U_C = W_{BC} = 800$ eV$= 1.3 \times 10^{-16}$ J,

$U_A - U_C = W_{AC} = 0$ eV $= 0$ J

57. 4.5×10^5 C, (6.3 or 6.4) $\times 10^8$ volt

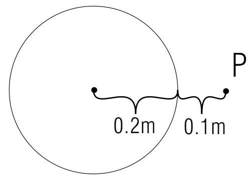
MODEL EXAM

USEFUL INFORMATION

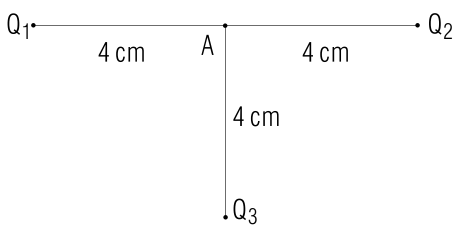
$k_e = 9.0 \times 10^9$ newton meter^2/coulomb^2

1. **Potential due to a uniformly-charged sphere.**

   The sphere shown in the figure has a radius 0.20 meter and has a charge -2.0×10^{-7} C uniformly distributed over its surface. The point P is 0.10 meter from the surface of the sphere. What is the electric potential at P?

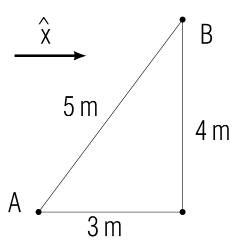


2. **Potential due to three charged particles.** The charged particles Q_1, Q_2, and Q_3 shown in the figure below individually produce electric potentials at point A of $+200$ volt, $+300$ volt, and -400 volt, respectively. What is the electric potential at A?



3. **Relation between work and electric field.**

   A particle with charge 4.0×10^{-10} C moves from point A to point B in an electric field, as shown in the figure below. The work done on the particle by the electric force during this motion is -6.0×10^{-8} J The electric field in this region is parallel to the unit vector $\hat{x}$.

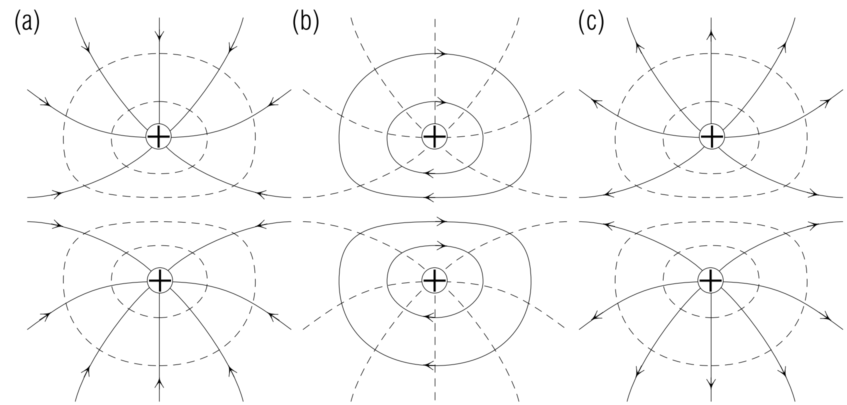


   a. What is the magnitude of the potential drop from point A to point B?

   b. Is the direction of $\vec{E}$ along $\hat{x}$ or opposite to $\hat{x}$?

   c. What is the magnitude of $\vec{E}$?

4. **Equipotential and field lines near two identically charged particles.** Which of the diagrams below best describes the electric field lines (solid lines) and the equipotential lines (dotted lines) due to two particles with identical charges of $+3 \times 10^{-10}$ C?



Brief Answers:

1. -6.0×10^3 volt
2. 100 volt
3. a. 150 volt; b. opposite; c. 50 volt/meter
4. (c)