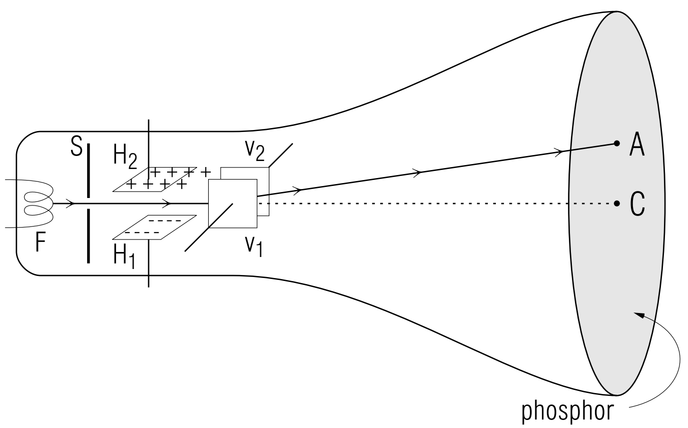
PHYSNET

MISN-0-411

ELECTRIC FORCE AND CHARGED PARTICLES



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

ELECTRIC FORCE AND CHARGED PARTICLES

by

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CONTENTS

Title: **Electric Force and Charged Particles**

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Input Skills:

1. State two properties of two-particle forces (MISN-0-408).
2. State the equation of motion for a particle (MISN-0-408).

Output Skills (Knowledge):

K1. Vocabulary: charge, electric insulator (or dielectric), electric conductor.

K2. State Coulomb's force law.

K3. State the principles of quantization and conservation of charge.

K4. State the charge and the mass of an electron.

K5. Describe the two methods of charge transfer.

Output Skills (Problem Solving):

S1. Given the charge distribution of nearby objects, qualitatively determine the electric force on a charged particle and the motion of this particle.

S2. Given the masses and charges of two interacting particles, relate the distance between them to their accelerations.

S3. Given a description of the initial charges on conductors or insulators, describe qualitatively the charge distribution resulting from their interaction.

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ELECTRIC FORCE AND CHARGED PARTICLES

Abstract:
The electric force is a fundamental force of the utmost importance. Since it is responsible for the interaction between atoms and molecules, it accounts ultimately for most phenomena studied in physics and for all phenomena studied in chemistry or biology. Furthermore, it leads to an enormous range of practical applications in all technology and instrumentation. The present unit will discuss some of the most basic properties and implications of the electric force. This discussion will pave the way for a much more extensive study of electric phenomena later in the book.

SECT.

A ELECTRIC CHARGE

OBSERVATION OF ELECTRIC FORCES

After a plastic comb is rubbed against fur, or is passed through dry hair, it acquires the property of attracting small pieces of paper. When such a comb is wiped against two light balls, such as ping-pong balls, these balls repel each other. (See Fig. A-1a). But if the comb is wiped against one of the balls and the fur is wiped against the other ball, the balls attract each other. (See Fig. A-1b.) These repulsive or attractive forces are apparent even if the balls are separated by an appreciable distance. Similar experiments can be performed with other materials, e.g., by rubbing a glass rod with a piece of silk and using balls wiped against these materials. The observed effects are said to be due to "frictional electricity."

These simple experiments lead to the important conclusion that two particles can interact by a long-range force having these properties: (1) This force may be either repulsive or attractive (unlike the gravitational force which is always attractive.) (2) This force can have a much larger magnitude than the gravitational force (since the gravitational force between two ping-pong balls would be immeasurably small).

Hence we conclude that the force appearing in the preceding experiments is a new kind of force which we shall call the "electric force." We can now ask this question: Under what conditions does this force appear and what are its basic properties?

DEFINITION OF ELECTRIC CHARGE

A particle is said to be "charged" if it has the property that it can experience an electric force due to some other particle. To define this property more precisely, suppose that several such charged particles (e.g., various ping-pong balls) can successively be placed at some point P which is at a fixed distance from a specified fixed charged particle A, such as a charged aluminum sphere. (See Fig. A-2.) Then the electric force $\vec{F}$ on any particle placed at P can readily be measured, e.g., by determining the deformation of a spring holding the particle at rest at P. (Note that this force must be directed either toward or away from A.) If one then

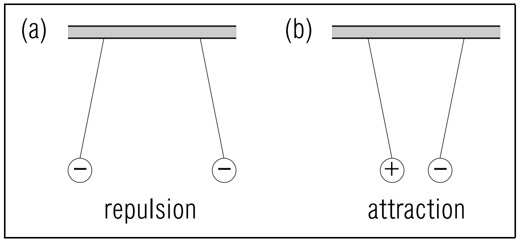


Fig. A-1: Experiment demonstrating mutual electric forces on two ping-pong balls suspended by threads.

measures and compares the electric forces $\vec{F}_1$ and $\vec{F}_2$ on any two particles 1 and 2 successively placed at P, one finds that the relative magnitude F_1/F_2 of the forces is always the same, irrespective of the position of P or of the nature of the charged object A. Furthermore, the relative direction of $\vec{F}_1$ and $\vec{F}_2$ (either the same or opposite) is also the same, irrespective of the position of P or of the nature of A. In short, the relative magnitude and direction of $\vec{F}_1$ and $\vec{F}_2$ depends only on the properties of the particles 1 and 2.

This conclusion suggests that each particle can be described by a property called "charge" and specified by a number q. The *magnitudes* of the charges of two particles are defined so that they are proportional to the magnitudes of the electric forces on these particles located at the same point, i.e.,

$$\frac{|q_1|}{|q_2|} = \frac{F_1}{F_2} \tag{A-1}$$

Furthermore, the signs of the charges of the particles are defined so that:

q_1 and q_2 have same signs if $\vec{F}_1$ and $\vec{F}_2$ have same direction, and opposite signs if $\vec{F}_1$ and $\vec{F}_2$ have opposite directions.	(A-2)

We can combine both Eq. (A-1) and Rule (A-2) by writing:

$$\frac{|q_1|}{|q_2|} = \pm\frac{F_1}{F_2} \tag{A-3}$$

where the plus sign applies if $\vec{F}_1$ and $\vec{F}_2$ have the same direction, and

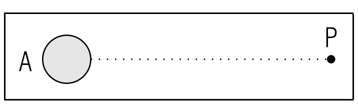


Fig. A-2: Experimental arrangement for comparing electric forces on various particles placed at the point P.

the minus sign applies if they have opposite directions. Thus we can summarize the meaning of charge by the following definition:

Def.	**Charge**: The charge q of a particle is a numerical quantity specifying the relative magnitude and direction of the electric force on the particle (so that $q_1/q_2 = \pm F_1/F_2$).	(A-4)

If the force $\vec{F}$ on a charged particle is due to *several* particles such as A, the superposition principle for force implies (as we shall show in detail in Unit 418) that the relation (A-3) remains valid for any two particles placed at the same point.

UNIT OF CHARGE

Like any other comparison procedure, the preceding definition of charge determines only the *ratio* of two charges. To specify a unique value for any *one* charge, it is then again convenient to compare all charges with one specific charge chosen as a standard. In the SI system this standard charge is called the "standard coulomb" and its value is indicated by the unit "coulomb" (abbreviated as "C"). The international convention specifying the choice of the standard coulomb will be discussed in Unit 427. [The "coulomb" is named in honor of Charles A. de Coulomb (1736-1806) who first investigated the properties of the electric force.]

The charge q_1 of any particle may then be measured in the arrangement of Fig. A-2 comparing it with the standard particle S having the standard charge $q_S = 1$ coulomb. Then the definition of charge, Eq. (A-3) or Def. (A-4), implies that

$$q_1 = \pm\frac{F_1}{F_S}q_S = \pm\frac{F_1}{F_S}\,\text{coulomb} \tag{A-5}$$

where the plus sign applies if the force $\vec{F}_1$ on the particle has the same direction as the force $\vec{F}_S$ on the standard coulomb, and the minus sign applies if $\vec{F}_1$ has the opposite direction. For example, if the force on the particle is 3 times as large and of opposite direction compared to the force $\vec{F}_S$ on the standard coulomb, the charge of the particle is equal to $q_1 = -3$ coulomb. (Actually a coulomb is such a large charge that ordinary objects rarely have charges exceeding 10^{-6} coulomb.)

CHARGE OF A SYSTEM

The charge Q of a *system* of particles is defined as its *total* charge, i.e., as the sum (with proper attention to signs) of the charges q_1, q_2, . . . of all the particles in the system.

Def. **Charge of a system**: $Q = q_1 + q_2 + \ldots$ (A-6)

Suppose that an object is small enough to be considered a particle, i.e., small enough so that all the constituent particles in the object are essentially located at the same point. Then the total electric force on such an object depends on its total charge in exactly the same way as the force on a single particle depends on its charge. *

> The electric force on a system will be discussed more fully in Unit 419.

Understanding the Definition of Charge (Cap. 1a)

A-1 *Statement and example:* A small ping-pong ball A is suspended at a point P near a charged comb. Because ball A has a charge q_A, it is acted on by an electric force $\vec{F}_A$ due to the comb. After removing ball A, a second ping-pong ball B of charge q_B is suspended at the same point P. It is acted on by an electric force $\vec{F}_B$ due to the comb. (a) Use the definition of charge to state the relation between these charges and forces. (b) Suppose both $\vec{F}_A$ and $\vec{F}_B$ are directed from P towards the comb, and have the magnitudes $F_A = 0.01\,\mathrm{N}$ and $F_B = 0.002\,\mathrm{N}$. If ball A has a charge of 5×10^{-9} coulomb, what is the charge of ball B? (c) Suppose $\vec{F}_A$ and $\vec{F}_B$ have the magnitudes given in part (b), but their directions are opposite to each other. What then is the charge of ball B? *(Answer: 104)*

A-2 *Properties of charge and mass:* (a) Is the quantity *charge* a number or a vector? If it is a number, what are its possible signs? What is its SI unit? (b) Answer the preceding questions about the quantity *mass*. (c) Which of the magnitudes,10^{-8} coulomb, 1 coulomb, or 10^8 coulomb, might be the charge of an ordinary object such as a comb run through dry hair? *(Answer: 109)*

A-3 *Comparison of electric and gravitational forces:* Consider two ping-pong balls which hang from threads as in Fig. A-1a. Give two reasons why these balls must interact by the electric force, rather than only by the gravitational force. *(Answer: 101)*

A-4 *Dependence of electric force on charge:* In the process of "electrolysis," charged ions in a solution are separated by electric forces due to charged metal plates attached to the terminals of a battery. A copper ion at a point P in the solution is acted on by an electric force of $5.0 \times 10^{-17}\,\mathrm{N}$ to the right. (a) The charge of a chlorine ion has a magnitude half that of the copper ion and has a sign opposite to that of the copper ion. What is the electric force on the chlorine ion when it is at P? (b) What is the electric force on a silver ion (at P) which has a charge equal in magnitude but opposite in sign to the charge of the chlorine ion? *(Answer: 107)*

Knowing About Charge of a System

A-5 The charged particles in a carbonate ion consist of 30 protons, each of charge 1.6×10^{-19} coulomb, and 32 electrons, each of charge -1.6×10^{-19} coulomb. What is the charge of the carbonate ion? At a point P in a solution undergoing electrolysis, a proton is acted on by an electric force of $2.0 \times 10^{-17}\,\mathrm{N}$ towards the left. What is the electric force on the carbonate ion (considered as a particle) when it is at the point P? *(Answer: 105)*

SECT.

B COULOMB'S ELECTRIC FORCE

Consider two particles 1 and 2 (e.g., two ping-pong balls) which have charges q_1 and q_2. (See Fig. B-1.) How does the electric force on either particle depend on the properties of the particles and on the distance R between them?

As usual, the direction of the force $\vec{F}$ on each particle must be along the line joining the particles, i.e., either away from the other particle (repulsive) or toward the other particle (attractive). Simple experiments then lead to this conclusion relating the direction of $\vec{F}$ to the signs of the charges q_1 and q_2: (See Fig. B-2.)

> The direction of $\vec{F}$ is repulsive if the charges have the same sign, attractive if they have opposite signs. (B-1)

Because of the reciprocal relation between mutual forces, the *magnitude* of the electric force on either particle due to the other must have the *same* value $F = |\vec{F}_{1,2}| = |\vec{F}_{2,1}|$. From our discussion in the preceding section we know that the magnitude F of the force on the first particle should be proportional to the magnitude $|q_1|$ of its charge (e.g., if $|q_1|$ were 3 times as large, F should also be 3 times as large). Similarly, the magnitude F of the electric force on the second particle should also be proportional to the magnitude $|q_2|$ of its charge (e.g., if $|q_2|$ were 3 times as large, F should also be 3 times as large). If both $|q_1|$ and $|q_2|$ were 3 times as large, the magnitude F of the force on either particle due to the other should then be larger by the factor 3×3, i.e., F should be proportional to the *product* $|q_1||q_2|$ of the magnitudes of the two charges.

Furthermore, Coulomb's experiments showed that F depends on the distance R of the charged particles so as to be proportional to $(1/R^2)$. [For example, if the distance R were 3 times as large, F would be multiplied by $(1/3)^2 = 1/9$.] Hence the dependence of F on the charges of the particles

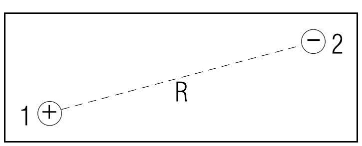


Fig. B-1: Electric interaction between two charged particles.

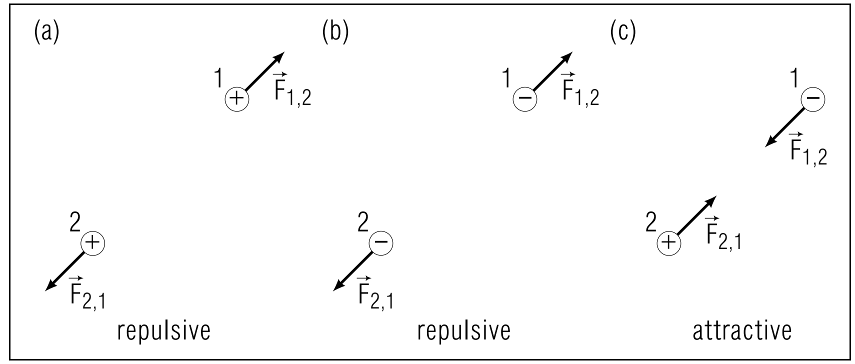


Fig. B-2: Electric forces on two interacting particles having charges of various signs.

and on the distance R between them can be summarized by writing

$$F = k_e \frac{|q_1|\,|q_2|}{R^2} \qquad \text{(B-2)}$$

where k_e is a constant which does *not* depend on the charges of the particles or on the distance between them. Thus k_e is a fundamental constant characterizing all electric interactions in our universe and called the "electric force constant."

The numerical value of the constant k_e can be determined by applying the relation (B-2) to a situation where all quantities, except k_e, can be directly measured. Thus one needs only to measure the magnitude F of the electric force in the case of two particles which have known charges and are separated by a known distance R. Such experimental measurements show that the value of k_e is

$$\boxed{k_e = 9.0 \times 10^9 \text{ newton meter}^2/\text{coulomb}^2\,.} \qquad \text{(B-3)}$$

(This value is sufficiently accurate for all our purposes. A more accurate value is listed in the appendix.) The units associated with k_e are such as to assure the consistency of the units in Eq. (B-2). Thus if the charges are expressed in terms of coulomb and the distance R in terms of meter, the force is expressed in terms of newton. *

> * In more advanced work the electric force constant k_e is often expressed in terms of another constant ϵ_0 defined so that $k_e = 1/(4\pi\epsilon_0)$ The use of the constant ϵ_0 simplifies some more advanced equations at the expense of complicating simple relations such as the force law Eq. (B-2).

DISCUSSION OF THE FORCE LAW

We can summarize all the preceding comments by this force law:

Coulomb's force law: $\vec{F}$ is repulsive if signs of charges are the same, attractive if signs of charges are opposite.

$$F = k_e |q_1|\,|q_2|/R^2 \qquad \text{(B-4)}$$

We shall call the electric force described by this force law the "Coulomb force" to distinguish it from more complex electric forces discussed later in the book.

Note that the Coulomb force, just like the gravitational force, depends on the distance R between the interacting particles proportionately to $1/R^2$. But these forces depend on different properties of the particles, the Coulomb force only on their charges and the gravitational force only on their masses.

Coulomb's law describes very accurately the electric interaction between two particles at rest relative to each other. Thus the fact that F is proportional to $1/R^2$ is confirmed by very precise experiments. * Furthermore, Coulomb's law remains valid even when the distance R between the interacting particles is as small as 10^{-15} meter, i.e., even when the Coulomb force becomes extremely large.

* These show that, if F were proportional to $1/R^n$, n could differ from 2 by no more than 10^{-16}.

If the interacting particles move relative to each other with a speed much less than the speed of light (3×10^8 meter/sec), Coulomb's law still describes their electric interaction quite adequately. On the other hand, when the particles move more rapidly, their interaction becomes more complex and gives rise to important new effects (such as magnetic effects) which will be discussed later in the book.

The Coulomb force on a charged particle due to *several* other charged particles can be simply found by the superposition principle. For example, in Fig. B-3 the *total* force F on the small charged ball B due to many

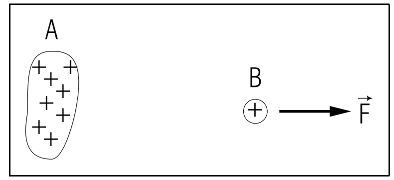


Fig. B-3: Total Coulomb force $\vec{F}$ on a small charged ball B due to many charged particles on an object A. (The + signs indicate individual positively charged particles.)

charged particles distributed over the large object A is merely the vector sum of the Coulomb forces on the ball due to all the charged particles on the object. This is called the "total Coulomb force" on the ball. It $\vec{F}$ is directed approximately away from the object since the ball is repelled by all the individual charged particles on the object.

Understanding Coulomb's Electric Force Law (Cap. 1b)

B-1 *Statement and example:* (a) Write an equation summarizing Coulomb's electric force law. (b) If the ping-pong balls shown in Fig. A-1a are separated by a distance of 3 cm, and each has a charge of -2×10^{-8} coulomb, what is the magnitude of the electric force on each ball due to the other? (c) If the ping-pong balls each have a mass of 2 gm, what is the magnitude of the gravitational force on each due to the earth? (d) Compare these electric and gravitational forces, by finding the ratio of the magnitude of the electric force on each ball divided by the magnitude of the gravitational force on the ball. *(Answer: 110)*

B-2 *Applicability:* An ion of charge q_1 is located at the point P, a distance R from the root tip having a charge q_2 (Fig. B-4). Why does Coulomb's electric force law *not* relate these quantities to the electric force on the ion due to the root tip? *(Answer: 102)*

B-3 *Comparison of relations:* A sodium ion of charge 1.6×10^{-19} coulomb is located at the point P shown in Fig. B-4. This ion is acted on by an electric force of 2×10^{-23} N $\hat{x}$ due to the root tip. What relation can be used to find the electric force due to the root tip on a sulfate ion of charge -3.2×10^{-19} coulomb (by using the information provided)? What is the value of this force? *(Answer: 113) (Suggestion: [s-10])*

B-4 *Finding values:* A sulfate ion in a solution is acted on by an electric force of -9.0×10^{-12} N $\hat{x}$ due to a nearby carbonate ion (Fig. B-5). If both these ions have charges of the same magnitude, and they are separated by a distance of 10 angstrom $= 10^{-9}$ meter, what is

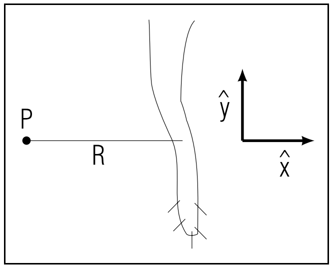


Fig. B-4.

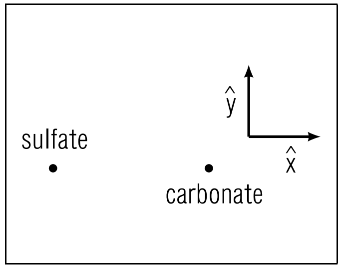


Fig. B-5.

the magnitude of the charge of each ion? Are the signs of these charges the same or opposite? *(Answer: 106)*

Describing Forces due to Charged Objects (Cap. 2)

B-5 Figure B-6 shows a possible device for removing polluting particles from gas rising through a smokestack. The particles acquire a negative charge as they pass the negatively charged needle at the bottom of the smokestack. Then the gas carries the particles upward between the charged plates 1 and 2. What are the approximate directions (right or left) of the electric force on a negatively charged particle due to plate 1 and due to plate 2? As this device operates, do polluting particles collect on plate 1 or on plate 2? *(Answer: 111) (Suggestion: [s-12])*

B-6 The two particles shown in Fig. B-7 have charges of the same magnitude but of opposite sign. What is the direction of the electric force on a positively charged particle placed at the point P midway between these particles? What is the direction of the electric force on this particle when it is placed at the point P'? *(Answer: 103) (Suggestion: [s-7])*

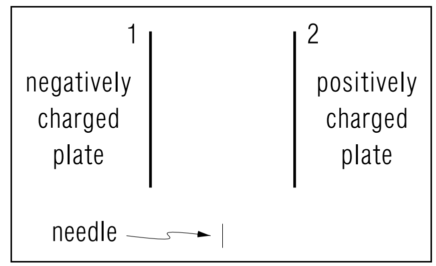


Fig. B-6.

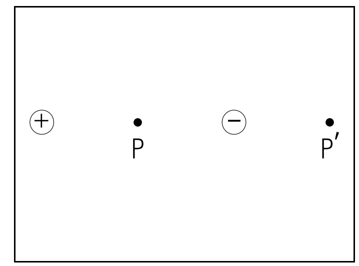


Fig. B-7.

SECT.

C ELECTRONS

The study of the structure of matter leads to the conclusion that all matter consists of a few kinds of atomic particles whose properties and interaction account for the observable properties of all objects. How can one experimentally obtain information about the most fundamental properties of such an atomic particle, its mass m and its charge q?

To determine the charge q of an atomic particle, one needs only compare the electric force $\vec{F}$ exerted on this particle by some charged object with the electric force $\vec{F}\prime$ exerted on some other particle having a known charge q' and located at the same position. Indeed, according to the definition of charge, Def. (A-4),

$$q = \pm \frac{F}{F'} q' \qquad \text{(C-1)}$$

where the sign is + or − depending on whether $\vec{F}$ and $\vec{F}\prime$ have the same or opposite directions.

In practice it is difficult to measure directly the force $\vec{F}$ on an atomic particle since one cannot weigh it or attach springs to it. But one can use observations of the motion of the particle to find its acceleration $\vec{a}$. Since $m\vec{a} = \vec{F}$, the relation (C-1) then implies that

$$q = \pm \frac{ma}{F'} q' \text{ or } \frac{q}{m} = \pm \frac{a}{F'} q' \qquad \text{(C-2)}$$

Hence measurements of the acceleration $\vec{a}$ of an atomic particle permit one to find q/m, the ratio of its charge q divided by its mass m.

CHARGE AND MASS OF THE ELECTRON

In a large variety of experiments one observes negatively charged particles having the *same* measured value of the ratio q/m. For example, such particles are observed as a result of disrupting (or "ionizing") the molecules of various gases, as a result of heating metals to very high temperatures, or as a result of illuminating certain metals with light. Since the negatively charged particles appearing in so many experiments are all characterized by the *same* value of q/m, one infers that these particles are contained in all matter and that they are all of the same

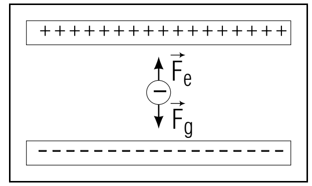


Fig. C-1: Millikan's oil drop experiment.

kind. Such a particle is called an "electron" and its measured value of q/m is

$$\frac{q}{m} = -1.76 \times 10^{11} \text{ coulomb/kilogram} \qquad \text{(C-3)}$$

In 1909 Robert Millikan (1868 - 1953) devised an ingenious method for determining the charge of the electron by a direct measurement of electric force. The method consists of spraying a few small oil droplets (about 10^{-6} meter in diameter) into the space between two metal plates. One or more electrons, obtained by disrupting the surrounding air with X-rays, may then become attached to such a droplet so that it acquires some charge Q. By placing appropriate charges on the metal plates, one can then maintain the charged droplet *at rest* between the plates. (See Fig. C-1.) This is called Milliken's Oil Drop Experiment. Under the conditions of this experiment, one knows that the total force on the droplet must be zero, i.e., that $\vec{F}_e + \vec{F}_g = 0$ where $\vec{F}_e$ is the upward electric force on the charged droplet due to the charges on the plates, and where $\vec{F}_g$ is the downward gravitational force on the droplet due to the earth. Thus a knowledge of the gravitational force $\vec{F}_g = M\vec{g}$ on the droplet can be used to find the electric force $\vec{F}_e$ on the droplet and thus to measure its charge Q. *

> * The mass M of the droplet can be determined by removing the charges on the plates and then observing how rapidly the droplet falls under the influence of gravity and the known frictional force due to the air.

The experiments show that the measured total charge Q on any droplet is always an integral multiple of some smallest charge q, i.e., that $Q = Nq$ where N is some integer. This result is expected if each electron has a charge q and there are N such electrons attached to the droplet. The measurement of q thus shows that the value of the charge of the electron is:

$$\boxed{q = -e, \text{ where } e = 1.60 \times 10^{-19}\,\text{coulomb}} \qquad \text{(C-4)}$$

Here we have used the conventional symbol e to denote the *magnitude* of the charge of the electron.

By combining this value of q with the experimental value Eq. (C-3) for the ratio q/m, we can immediately calculate the mass m of the electron. Thus we find that:

$$\boxed{m = 9.11 \times 10^{-31}\,\text{kilogram}} \qquad \text{(C-5)}$$

This mass is obviously very small.

Knowing About Electrons

C-1 An oil drop D of mass 8×10^{-16} kilogram is at rest at a point between the two charged plates used in the Millikan oil drop experiment. The drop interacts only with these plates and with the earth (Fig. C-1). (a) What is the gravitational force on this drop? (b) What is the electric force on the drop? (c) When a drop which has the known charge 4.8×10^{-19} coulomb is located at the point P, it is acted on by an electric force of 12×10^{-15} N downward. What is the charge q of the drop D? (d) How many ("excess") electrons make up the charge on the drop D? (e) A typical oil drop contains $N = 1 \times 10^{11}$ electrons. Compare the number n of "charge-q" electrons with N by expressing n as a number times N. *(Answer: 108) (Suggestion: [s-1])*

Relating Motion to the Electric Interaction (Cap. 3)

C-2 In a Van de Graaff accelerator, a charged sphere (which can be considered as a particle) exerts an electric force on ions, which are thus accelerated. (Accelerated ions have many biological and medical uses, including the destruction of diseased tissue.) In a typical situation, an ion of charge $q = 1.6 \times 10^{-19}$ coulomb and mass $m = 1.7 \times 10^{-27}$ kg is initially a distance $R = 0.40$ meter from the center of the sphere, which has a charge $Q = 2.5 \times 10^{-4}$ coulomb. (The ion is isolated from particles other than those in the sphere). (a) Express the magnitude a of the ion's initial acceleration in term of the symbols provided. (b) What is the value of a? (c) If the ion is initially at rest, does it move towards or away from the charged sphere? *(Answer: 114) (Suggestion: [s-6])*

SECT.

D QUANTIZATION OF CHARGE

Although every atom or molecule contains electrons, precise measurements show that its total charge is zero. Hence an atom or molecule must contain positively charged particles having a total positive charge equal in magnitude to the total negative charge of all its electrons.

For example, the hydrogen atom consists simply of one electron and one other fundamental particle called a "proton." The properties of the proton are very different from those of the electron; e.g., the mass of the proton is more than 1800 times as large as the mass of the electron. But since the total charge of the hydrogen atom is zero, the sum of the charge q_p of the proton and the charge $-e$ of the electron is $q_p + (-e) = q_p - e = 0$. Thus $q_p = e$ so that the proton has a positive charge *equal* in magnitude to that of the electron, although the other properties of the proton are very different. There are many other kinds of atomic particles. For example, the "nuclei" of more complicated atoms consist of protons and uncharged particles called "neutrons." In addition, there are other "elementary particles" (designated by names such as "mesons" or "hyperons") which have widely different masses and widely different "lifetimes" between their creation and their decay into more stable particles. Yet each of these particles has a charge q which is always an integral multiple of the charge e (so that $q = Ne$, where N is some integer which may be positive, negative, or zero). Thus we arrive at the following conclusion, called the principle of "quantization of charge":

Quantization of charge: Every charge must always be some integral multiple of e, where e is the magnitude of the charge of the electron.	(D-1)

Hence all charge always appears in the form of discrete packets (or "quanta") of magnitude e. For example, it is *not* possible to increase the total charge Q in an object by a small amount such as 10^{-20} coulomb, but only by increments of size $e = 1.60 \times 10^{-19}$ coulomb. When one is dealing with a large total charge Q (such as 10^{-6} coulomb) an increment of size e is negligibly small compared to Q so that the discrete nature of charge is not noticeable. But when one is dealing with a small total charge Q (such as 10^{-17} coulomb), the discrete size of the increment e is very apparent.

Knowing About Charge Quantization

D-1 Three students computing the charge of an unknown ion obtain the results: -3.0×10^{-19} coulomb, -4.8×10^{-19} coulomb, 5.0×10^{-19} coulomb. Which of these three results could be the correct charge of the ion? Briefly explain why the other results must be incorrect. *(Answer: 119)*

D-2 By using an instrument called an electrometer, an experimenter finds that the electric charge on a metal sphere is -3.256×10^{-8} coulomb. (a) How many excess electrons are on this sphere? (b) Suppose one more electron is added to the sphere. What then is the sphere's charge as measured by the electrometer? (Express your answer with the four significant figures indicating the precision of the electrometer's measurement.) Does the addition of one electron observably change the charge of the sphere? *(Answer: 116) (Suggestion: [s-2])*

SECT.

E CONSERVATION OF CHARGE

Suppose that every atomic particle has a charge unaffected by any interaction with other particles. If atomic particles are neither created nor destroyed, then this conclusion must be true:

Conservation of charge: If no charged particles pass between a region and its surroundings, the total charge inside this region remains constant.	(E-1)

This "principle of conservation of charge" is actually found to be *always* true, even under extreme conditions where particles *are* created or destroyed. For instance, energetic X-rays, which are uncharged, can create particle pairs consisting of an electron and a "positron," But the charge $q = e$ of the positron is then found to be opposite to the charge -e of the electron. Hence the *total* charge remains equal to zero as it was before creation of these particles.

Example E-1: Charge transfer

Suppose that an uncharged glass rod is rubbed against an uncharged piece of silk. Then the total charge in the region containing the rod and the silk is zero. *After* charged particles have been transferred between the rod and the silk, the total charge must then still be zero. Thus the charge Q_r on the rod and the charge Q_s on the silk must add to zero, i.e., $Q_r + Q_s = 0$. Hence $Q_r = -Q_s$ so that the rod and the silk have charges of equal magnitude but opposite signs.

Knowing About Conservation of Charge

E-1 A glass rod with an initial charge of -2.4×10^{-9} coulomb and an initially uncharged metal ball are in dry air and supported by wooden handles so that no charge can pass from these objects to their surroundings. (a) After the ball is touched with the glass rod, the ball's charge is -0.8×10^{-9} coulomb. What then is the charge of the rod? (b) During the process described, the charge of the ball changes, i.e., its charge is *not* conserved. Explain why the principle of conservation of charge does not apply to the system of the ball alone. *(Answer: 112)*

SECT.
F DEFLECTION OF CHARGED PARTICLES

Many practical applications use electric forces to deflect charged particles in desired ways. Let us describe several examples.

OSCILLOSCOPES AND TELEVISION TUBES

For many purposes a pen is too slow a writing device since it cannot be moved across a sheet of paper in much less than about 0.1 second. If we need a writing device capable of moving back and forth rapidly, it must be capable of moving with a large acceleration. But, for a given force $\vec{F} = m\vec{a}$ applied to such a device, the acceleration $\vec{a}$ of the device is larger if its mass m is smaller. Thus an electron should be usable as a very fast writing device since its mass is extremely small. Furthermore, the electron has a charge so that it can be easily accelerated by electric forces.

The preceding suggestion is exploited in an extremely useful an important instrument called the "oscilloscope." The essential component of this instrument is the oscilloscope tube, illustrated schematically in Fig. F-1. This is an evacuated glass tube in which electrons are emitted from a metal filament F heated to a high temperature. The electrons are then accelerated by electric forces toward S and then emerge as a fast-moving beam of electrons traveling under the influence of no further forces. Hence the electrons continue to travel along a straight line until they hit the center C of the face of the oscilloscope tube. This face is covered with a "phosphor," a substance (such as zinc sulfide) which emits light from any spot where it is struck by fast electrons. Thus the point where the electron beam hits the face of the oscilloscope is readily visible.

The electrons can be deflected from their straight path if they are acted on by an electric force while between the pair of horizontal deflecting plates H_1 and H_2. In Fig. F-1, a transfer of charge from the upper to the lower plate makes the lower plate negatively charged and the upper plate positively charged. The electric force on the electron beam between the plates is then in the upward direction. Hence this beam is deflected upward so as to strike the face of the tube at some point A above its center C. A larger amount of charge transferred between the deflecting plates would result in a correspondingly larger deflection of the electron beam. A transfer of charge between the plates in the *opposite* direction

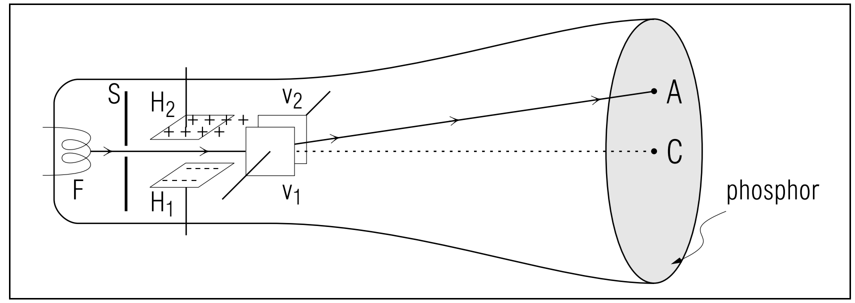


Fig. F-1.

would result in a corresponding deflection of the beam *below* the center C of the face of the tube. Thus a transfer charge between the horizontal deflecting plates results in a vertical deflection of the electron beam and makes it possible to trace out a visible vertical line on the face of the oscilloscope tube.

Another pair of vertical deflecting plates V_1 and V_2 is placed after the horizontal deflecting plates H_1 and H_2. A transfer of charge between these vertical plates can be used to deflect the beam in a horizontal direction. The two pairs of plates together can then deflect the beam by any vertical and horizontal amount so as to produce a spot of light at any point on the face of the oscilloscope tube.

Since electrons have such a small mass, both the transfer of charge (i.e., of electrons) between the plates and the resulting deflection of the electron beam can be achieved extremely rapidly. Thus the oscilloscope tube is an extremely rapid writing instrument, allowing one to display in immediate graphical form the relationship between any two variables whose measurement can be reduced to electrical form. Hence oscilloscopes are commonly found in almost any laboratory. For example, the oscilloscope can be used to display the electrical signals from nerve impulses in a physiology laboratory, or to display a patient's heart beat and other vital signs in the intensive care unit of a hospital.

A television tube is essentially an oscilloscope tube in which the electron beam is used to form a picture on the face of the tube. This is done by letting the electron beam trace out about 600 adjacent horizontal lines while varying its intensity so as to produce points of differing light intensity on the face of the tube. The bright and dark spots thus produced form a picture which must be repeated (or modified) about 30 times per second in order to be perceived by the eye as the continuous transforma-

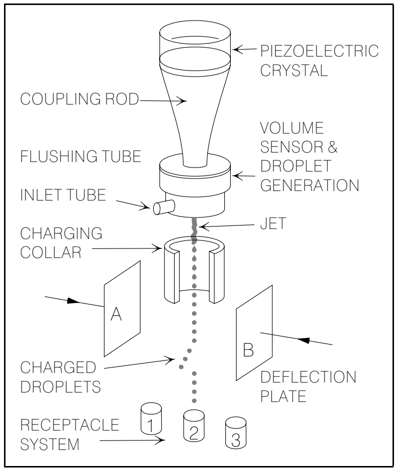


Fig. F-2: Separation of biological cells by electric deflection of charged droplets (the diagram is adapted from M. J. Fulwyler, *Science*, vol. 150, p. 910, 1965, and *Review of Scientific Instruments*, vol. 140, p. 42, 1969).

tion of a visual image. In order to trace out about 600 lines every 1/30th of a second, the electron beam must be able to traverse the face of the oscilloscope in a time of about 10^{-6} second. Only a writing instrument which exploits the small mass of the electron can achieve such high writing speeds.

DEFLECTION OF DROPLETS AND SEPARATION OF BIOLOGICAL CELLS

Charged droplets of a liquid can be deflected in a manner similar to that used for electrons in an oscilloscope. For example, suppose that a dilute solution contains biological cells with different properties (such as size or chemical composition) and that one wishes to separate these cells in order to study cells of a particular kind. To achieve this goal, one can use this procedure: One lets the solution pass through a nozzle designed to break the emerging stream of liquid into small droplets which are then charged (See Fig. F-2.) The properties of a cell in an emerging droplet are immediately ascertained by suitable instrumentation (e.g., cell size may be determined by electrical measurements or chemical composition by light absorption). By adjusting appropriately the charges on a pair of deflection plates, each such droplet is then deflected into a receptacle reserved for cells of a particular type. (This separation process does not kill the cells.)

Relating Particle Motion to Nearby Charge Distributions (Cap. 2)

$\boxed{\text{F-1}}$ Suppose the liquid droplets produced by the cell sorter shown in Fig. F-2 have a positive charge. When the sorter is operating the charges of plates A and B have charges with equal magnitude but opposite in sign. When both of these charges are zero, the droplets move directly downward, into container 2. (a) If plate A is negatively charged and plate B positively charged, into which of the three containers will the droplet be deflected? (b) Suppose that the charges of plates A and B are such that the droplets are deflected into container 3. Then a container 4 is added to the right of container 3. In order to deflect the droplets into container 4, should the magnitude of the charges on plates A and B be made larger or smaller? Should the sign of the charge on plate B remain the same or changed to the opposite sign? *(Answer: 123) (Suggestion: [s-15])*

$\boxed{\text{F-2}}$ In an oscilloscope (e.g.) the one shown in Fig. F-1) each pair of deflecting plates (such as V_1 and V_2) has charges of equal magnitude but opposite sign. What must be the sign of the charge on each of the four plates when the beam of electrons produces on the screen a spot located below and to the left of the center C (when viewed from the right)? *(Answer: 118) (Suggestion: [s-14])*

SECT.

G SCATTERING AND THE ATOMIC NUCLEUS

Suppose that some things (e.g., particles or light) are emitted from a source and arrive at a detector after being affected by interaction with some intervening object. Then we say the things were "scattered" by the object. The observed scattering may provide valuable information about the object. For example, much of the information about the world around us comes from observations where light from the sun (a source) arrives at our eyes (detectors) after being affected (scattered) by intervening objects. Thus we "see" the objects and can make inferences about them.

To obtain information about a charged object, one can scatter from it charged particles (since appreciable scattering is then produced by the electric interaction). Our basic knowledge of the structure of the atom comes from such scattering experiments, first performed around 1912 by Ernest Rutherford (1871-1927). As illustrated in Fig. G-1, he used positively charged "alpha particles" (helium nuclei), emitted by a radioactive source, to bombard a thin foil of metal. Alpha particles scattered by the atoms in the metal could then be detected by flashes of light produced when the alpha particles struck a phosphor-coated screen.

Since the total charge of an atom is zero, the atom must contain positively charged particles (protons) having a total positive charge equal in magnitude to the total negative charge of all the electrons in the atom. Suppose that all these charged particles were distributed at random throughout the atom, like raisins in a cake. Then an alpha particle incident on such an atom would be slightly deflected in different directions as it alternately passes close to positively and negatively charged particles in the atom. The result would thus be merely a small net deflection of the alpha particle.

The experimental results are entirely different from these expectations. In fact, some alpha particles are scattered through very large angles (so large that a few alpha particles are nearly scattered back along the direction from which they came). To account for these data, one is led to the conclusion that all the positively charged particles in an atom are concentrated in a very small region called the atomic "nucleus." (See Fig. G-2.) Then the large positive charge concentrated in such a nucleus exerts a large electric force on an alpha particle passing close to the nucleus and thus deflects the alpha particle through a large angle. Indeed,

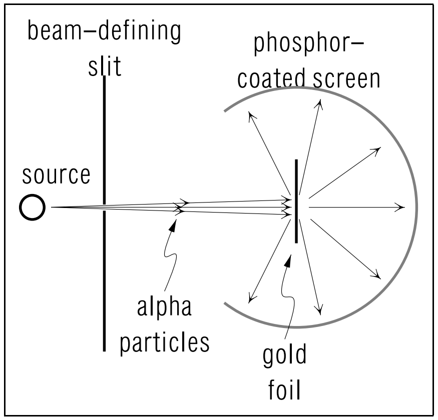


Fig. G-1: Rutherford's experiment for observing the scattering of alpha particles by gold atoms.

all experimental data can be explained quantitatively if one assumes that the positively charged nucleus has a size of the order of 10^{-15} meter (i.e., very much smaller than the atomic size which is about 10^{-10} meter). (To account for the fact that the negatively charged electrons do not simply fall into the positively charged nucleus, one can imagine that the electrons revolve around the nucleus in the same way as the planets revolve around the sun. This planetary model of the atom is, however, not quite adequate and will be revised in later units of the book.)

ELECTRON MICROSCOPE

The electron microscope obtains detailed information about a specimen by using the scattering of electrons by atoms in the specimen. In such a microscope electrons pass through a very thin specimen (such as a slice of biological material 10^{-7} meter thick) and finally strike a phosphor where they produce light. (As shown in Fig. G-3a, the electrons, specimen,

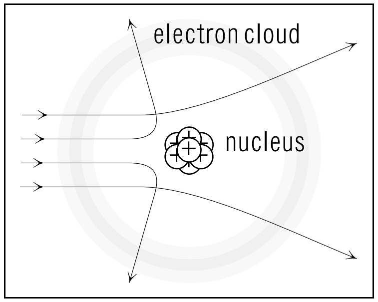


Fig. G-2: Paths of charged alpha particles scattered by the nucleus of an atom.

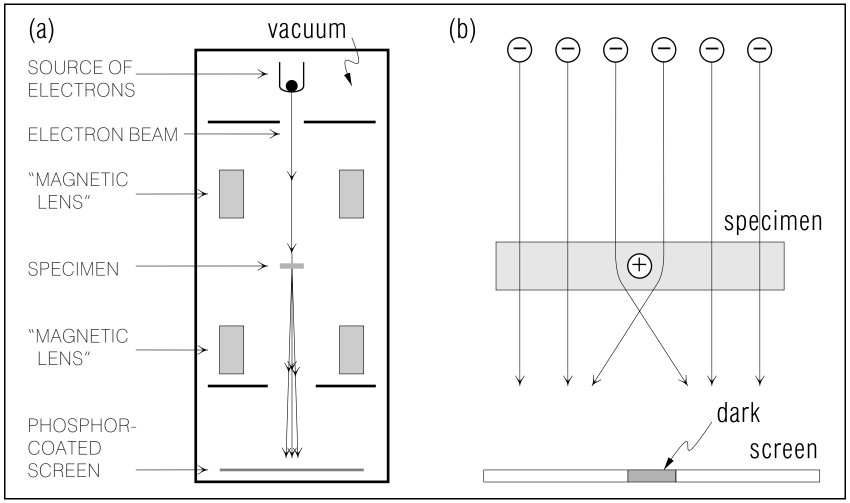


Fig. G-3: Electron microscope. (a) Essential components of the microscope. (b) Grossly magnified view of the specimen, showing scattering of electrons by a nucleus in the specimen.

and phosphor are all contained in an evacuated enclosure to prevent the scattering of electrons by molecules of air.) An electron passing close to an atomic nucleus in the specimen is deflected from its normal straight path. Hence fewer electrons emerge on the other side of the specimen wherever it contains nuclei which scatter electrons appreciably. (See Fig. G-3b.) Correspondingly, the reduced number of electrons striking the phosphor results in a darker region on the phosphor, i.e., the scattering of electrons by a nucleus in the specimen results in a shadow on the phosphor-coated screen. (The shadow produced on this screen is vastly larger than would be produced on a screen placed just behind the specimen because the electron microscope is cleverly designed to deflect electrons by magnetic forces so as to produce a magnified image of the specimen.)

Biological specimens consist mostly of light atoms (such as hydrogen, carbon, nitrogen, phosphorus, ...) which contain only few electrons and whose nuclei contain thus only small positive charges. As a result, such nuclei produce such small electron scattering that no distinct shadows appear on the screen of the electron microscope. To remedy this situation, one usually "stains" the specimen before insertion in the electron microscope. This can be done by immersing the specimen in a solution of molecules which attach themselves selectively to certain parts of the

specimen (such as to proteins or nucleic acids) and which contain heavy atomic nuclei (such as lead or uranium) whose large charges produce large electron scattering. For example, chemicals such as lead citrate or uranyl acetate are commonly used for the staining of biological specimens. When a stained biological cell is placed in the electron microscope, the image produced on the phosphor-coated screen is then due to the scattering of electrons from the highly charged nuclei attached to selected sites of the cell. To interpret this image, one must make inferences about the structure of the cell which, after staining, gives rise to the electron scattering producing the observed image. Needless to say, the task of proper interpretation may be difficult.

Knowing About Electron Microscope Images

G-1 The elements carbon (C), chlorine (Cl), hydrogen (H), oxygen (O) nitrogen (N), and phosphorus (P) consist of light atoms and their nuclei have relatively small charges. In contrast, lead (Pb), Osmium (Os), and tungsten (W) consist of heavy atoms with nuclei of much larger charge. All of the following compounds are used as stains to produce contrast in specimens to be observed with a microscope. Which compounds are used in light microscopy, and which in electron microscopy? (a) $C_{16}H_{18}N_3SCl$ (b) OsO_4 (c) $Pb_3(PO_4)_2$ (d) $C_6H_5CH[C_6H_4N(CH_3)_2]_2$ (e) $H_3PW_{12}O_{40}\cdot 14H_2O$ (f) $[(CH_3)_2NC_6H_4]_3COH$ *(Answer: 115)*

Relating Particle Motion to Nearby Charge Distributions (Cap. 2)

G-2 Figure G-4 shows two possible paths for a particle scattered by an atomic nucleus. (a) Which drawing best describes the path of an electron scattered by a nucleus in a biological sample? (b) Which drawing best describes the path of a positively charged alpha particle scattered by a gold nucleus in the Rutherford scattering experiment? *(Answer: 121)*

Relating Motion to the Electric Interaction (Cap. 3)

G-3 According to a simple "planetary" model, a hydrogen atom consists of an electron (of mass m_e and charge q_e) moving with constant speed along a circular path of radius r around a proton (of mass m_p and charge q_p) which remains at rest relative to an inertial frame. (a) Write an expression for the magnitude a of the electron's acceleration in

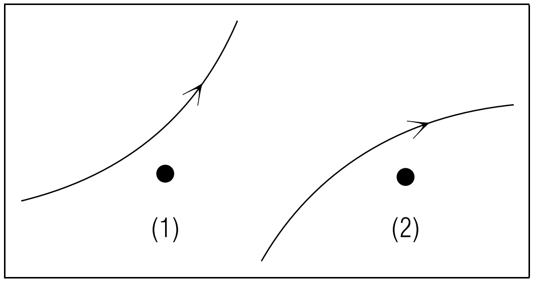


Fig. G-4.

terms of symbols for known quantities. (b) The electron and proton both have a charge of magnitude 1.6×10^{-19} coulomb, and they are separated by a distance of 0.5 angstrom $= 5 \times 10^{-11}$ meter. The proton has a mass of 1.7×10^{-27} kg, and the electron has a mass of 9×10^{-31} kg. According to this model what is the magnitude a of the electron's acceleration? (c) What is the corresponding speed of the electron as described in this model? *(Answer: 127) (Suggestion: [s-3])*

SECT. H ELECTRIC PROPERTIES OF MATTER

Large-scale objects, such as baseballs or persons, consist of enormously many atoms and contain thus enormously many charged particles (electrons and protons). Such an object is, however, ordinarily "uncharged" or "electrically neutral" (i.e., its total charge is zero) because the total charge of the positively charged particles in the object is equal in magnitude to the total charge of the negatively charged particles in the object. But when an object gains or loses some charged particles by transfer from some other object, it may acquire a slight excess of charge of either sign. Then the total charge of the object is no longer equal to zero and the object is said to be "charged."

To facilitate the qualitative discussion of charged objects, we shall use diagrams which indicate positive charges by + signs and negative charges by − signs. (See Fig. H-1.) Each + sign is intended to represent a *net* amount of positive charge located near the + sign. (This charge is the excess positive charge due to very many charged atomic particles, of both signs, located in the region near the + sign.) Similarly, each − sign is intended to represent a *net* amount of negative charge located near the − sign. (This charge is the excess negative charge due to very many charged atomic particles, of both signs, located in the region near the − sign.) The charged atomic particles in a material may be nearly fixed in position or fairly free to move. Accordingly, it is useful to distinguish qualitatively between the following two classes of materials.

Def. | **Electric insulator (or dielectric)**: A material in which charged atomic particles can move only slightly from their normal positions. | (H-1)

Def. | **Electric conductor**: A material containing charged atomic particles which can move throughout the entire material. | (H-2)

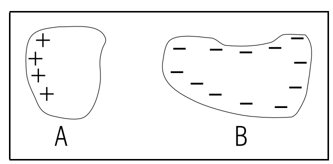


Fig. H-1: Charge distributions on two charged objects. (In this diagram, *A* is an insulator and *B* a conductor.)

Materials such as plastics, rubber, or oil are good insulators. If charged atomic particles are placed on an object consisting of such an insulating material, these charged particles will remain where they have been placed and will only move very slightly as a result of any electric forces.

The mobile charged atomic particles in a conductor may be either electrons or "ions" (i.e., atoms which have lost or gained one or more electrons). Metals are good conductors because they contain electrons which are free to move (although the remaining positive ions are fixed in position). Electrolytic solutions (such as solutions of NaCl in water) are conductors because they contain positive and negative ions which are free to move (e.g., Na^+ and Cl^- ions).

Although the mobile charged particles in a conductor are free to move throughout the entire conductor, they are ordinarily confined within the conductor by forces which prevent their escape through its surface. The mobile charged particles can, however, move freely between two conductors which are in good contact with each other.

The mobile charged particles in any isolated set of conductors will continue to move, and collide with the stationary atoms, until they come to rest (more precisely, until their *average* velocity is, except for fluctuations, reduced to zero). In this final situation, called an "equilibrium" situation, the charged particles must then be distributed throughout the conductors in such a way that the *total* force on every such particle is zero (since this assures that such a particle does not tend to move).

Since like charges repel each other, the mobile charged particles in an isolated conductor tend to move as far away from each other as possible. In the final equilibrium situation, they tend therefore to be spread out over the entire *surface* of the conductor. For example, if extra electrons are *added* to an isolated metal object, these electrons repel each other and thus spread out over the entire surface of the metal. Hence the total negative charge of the metal becomes distributed over its surface, as illustrated in Fig. H-2a. Similarly, if electrons are *removed* from the metal, the remaining electrons are attracted toward the positive ions in the inside of the metal so that an excess number of positive ions are left near the surface. Hence the total positive charge of the metal becomes distributed over its surface, as illustrated in Fig. H-2b. (The removal of the negatively charged electrons has thus the same effect as the addition of mobile positively charged particles.)

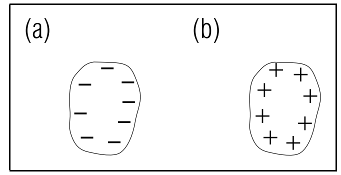


Fig. H-2: Charge distribution on a metal (a) after the addition of extra electrons and (b) after the removal of some electrons.

The spreading of charges over the entire surface of a conductor can be made evident if a part of this conductor is movable. For example, Fig. H-3 shows a metal rod to which there is attached a thin metal foil hinged at its top so that it is free to move. Any electric charge transferred to the rod then becomes distributed over the entire surface of the conductor, in particular, also over the bottom of the rod and the attached foil. Since all these charges have the same sign, the mutual repulsion between the bottom of the rod and the foil then causes the foil to swing away from the rod by some angle θ. This apparatus is called an electroscope.

Because this angle increases as the amount of charge transferred to the rod increases, the experimental arrangement in Fig. H-3 (called an "electroscope") can be used to determine the charge on the rod by measurements of the angle θ.

Knowing About Electric Properties of Matter

H-1 *Symbols for charge distributions:* The electric eel (phElectrophorus) has organs near its head and tail which produce the charge distribution indicated in Fig. H-4 (this charge distribution can cause a flow of charge through another fish, stunning the fish and making it easy prey). Which of the following statements correctly describe the charge distribution represented by Fig. H-4? (a) The eel's tail region consists

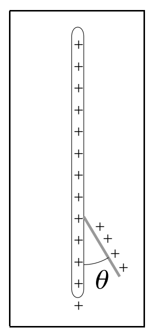


Fig. H-3: An "electroscope" consisting of a metal foil hinged to a metal rod.

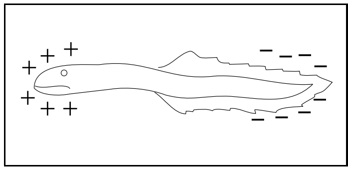


Fig. H-4.

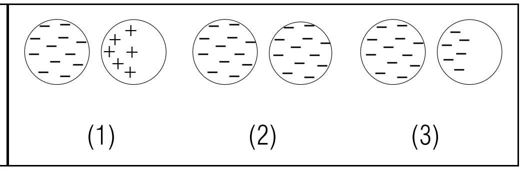


Fig. H-5.

almost entirely of electrons. (b) There is a net excess of positive charge near the eel's head. (c) The number of excess electrons in the tail region is very small compared with the total number of electrons in the eel. *(Answer: 124) (Suggestion: [s-5])*

H-2 *Conductors and insulators:* Two spheres, one initially uncharged, and one with an initial negative charge, are supported on insulating handles so that no charge can enter or leave the two spheres. The two spheres are touched together and then separated. Which of the drawings in Fig. H-5 shows a final possible charge distribution of the two spheres (a) if they are both conductors, and (b) if they are both insulators? *(Answer: 117) (Suggestion: [s-8])*

Qualitatively Describing Charge Distributions (Cap. 4)

H-3 (a) Which of the electroscopes shown in Fig. H-6 has the charge which is largest in magnitude? (b) Which electroscope has a charge which is zero (or of very small magnitude)? (c) Either find the sign of the charge on each electroscope, or explain why this sign cannot be found? *(Answer: 120) (Suggestion: [s-11])*

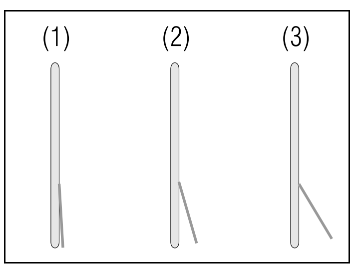


Fig. H-6.

SECT. I

CHARGE TRANSFER

CHARGING BY CONTACT

Suppose that some object A, having a total charge Q_A is brought into contact with some other object B, having a total charge Q_B. Then some charged atomic particles may move from one of these objects to the other, i.e., there may occur some charge transfer between the objects. As a result, the objects have afterwards total charges Q'_A and Q'_B which are ordinarily different from their original charges. Because of the conservation of charge, the total charge on *both* objects remains, however, the same so that

$$Q'_A + Q'_B = Q_A + Q_B \tag{I-1}$$

If the objects are insulators, little charge is transferred between them when they are touched to each other. But if the objects are conductors, charge is ordinarily transferred from one to the other whenever they are brought into contact. Indeed, suppose that the metal conductors A and B are in contact. Then they form a single composite conductor (consisting of A and B) since the electrons can move freely between the two conductors. But, as we know from our discussion in the preceding section, the motion of the electrons results in a redistribution of the total charge over the entire surface of this composite conductor and thus ordinarily in the transfer of some charge from A to B.

Suppose that initially the conductor A has a total charge Q_A while the conductor B is uncharged so that $Q_B = 0$. If the conductors are brought into contact, some net charge will then be transferred from A to B. (See Fig. I-1.) The result is that the conductor B becomes charged while the conductor A becomes partially discharged (since A is left with a final total charge having a *smaller* magnitude than its initial charge).

If the conductor A is brought into contact with some uncharged conductor B which is much larger than A (e.g., with the earth), the redistribution of charges over the entire surface of both conductors leaves almost no charge on A and almost all the charge on B. Hence A is left almost totally discharged with a final total charge Q'_A which is nearly zero. (See Fig. I-2.) For example, if a person picks up a charged metal object with

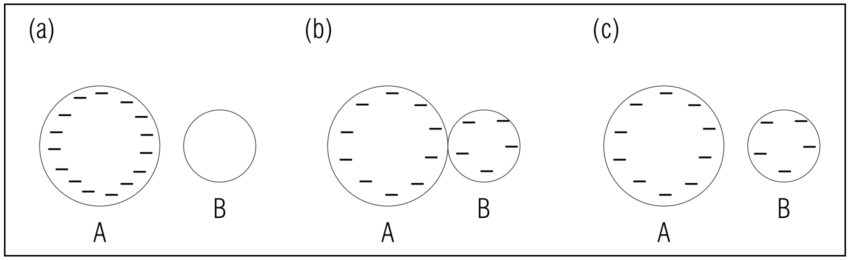


Fig. I-1: Charging of a conductor B by contact with a charged conductor A.

his bare hand, the object becomes almost discharged because most of its charge passes into the person (who is a good conductor). To avoid discharging the object, the person must thus pick it up by using a handle or glove made of electrically insulating material.

CHARGING BY INDUCTION

Suppose that a charged object A (insulating or conducting) is brought near to an uncharged metal object B *without* touching it. (See Fig. I-3.) Then all the charged particles in A produce some total electric force on every charged particle in B and thus cause the mobile electrons in B to move. For example, if A is negatively charged, the electrons in B are repelled from A. Hence these electrons move toward the side of B further from A and thus cause this side to become negatively charged. At the same time the side of B closer to A is left with a deficiency of electrons and thus becomes positively charged. The charges which have thus become separated in the object B are called "induced" charges resulting from the

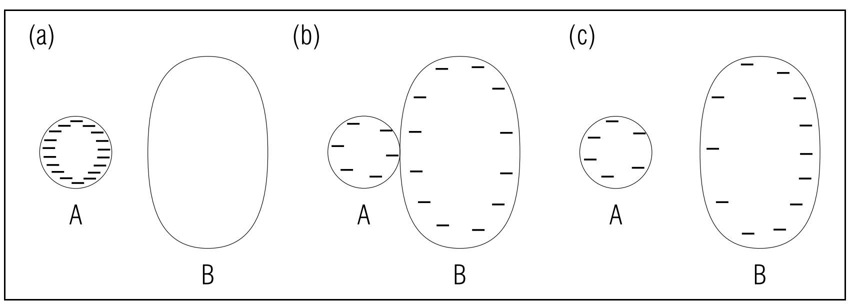


Fig. I-2: Discharging of a conductor A by contact with a large conductor B.

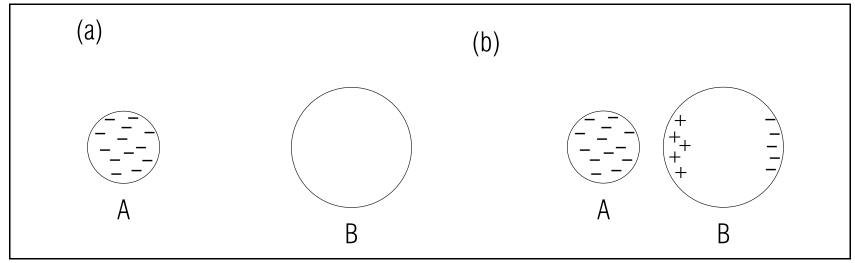


Fig. I-3: Charges induced in a conductor B by a charged object A brought near to B.

proximity of the charged object A. *

* The mutual attraction between the oppositely charged particles in B opposes their separation. Hence the extent of charge separation in B remains relatively small, but increases as A is brought nearer to B.

The separation of charges produced in a conductor by the proximity of a charged object can be used to charge a conductor *without* any contact with the charged object. It is only necessary to segregate the separated charges permanently by separating the conductor into two pieces. To do this in practice, we start with a conductor which consists of two separate conductors B and C which touch each other and which are originally uncharged. (See Fig. I-4a.) Suppose that a charged object A is brought close to the joined conductors B and C, as shown in Fig. I-4b. If A is negatively charged, the charges on the joined conductors become then separated so that the conductor C further from A acquires a net negative charge and so that the conductor B closer to A acquires a net positive charge. The conductors B and C are then separated from each other, as shown in Fig. I-4c. Finally, the charged object A is removed, as shown in Fig. I-4d. The net result of the preceding steps is that both the conductors B and C have become charged, although neither has ever touched the charged object A. We then say that the conductors B and C have been charged "by induction."

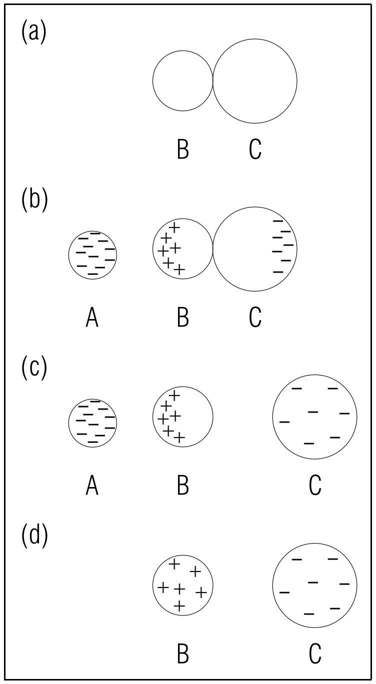


Fig. I-4: Charging by induction.

Knowing About Conductors and Insulators

I-1 To determine whether a substance is a conductor or an insulator, a sample of the substance is placed between a negatively charged battery terminal and an initially uncharged conducting metal ball. After waiting a few seconds, the objects are separated, and the charge of the ball is measured. The table below lists the results of this experiment for several substances. Which of these substances are insulators and which are conductors? *(Answer: 125)*

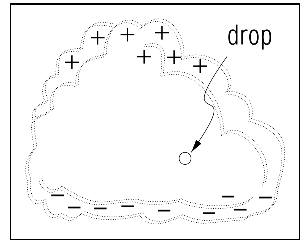


Fig. I-5.

Substance	*Resulting charge of metal ball*
glass	zero
human body	negative
rubber	zero
distilled water	zero
salt water	negative
graphite	negative

Qualitatively Describing Charge Distributions (Cap. 4)

I-2 Anesthetic gases and oxygen can easily be ignited by sparks resulting from any charged objects in an operating room. A surgeon walking down a carpeted hallway, has initially a positive charge. As he steps on the conducting floor of the operating room, does the surgeon's charge increase, decrease, or remain the same (a) if he is wearing rubber soled street shoes, (b) if he is wearing operating room shoes constructed with graphite rods extending through the soles? *(Answer: 128) (Suggestion: [s-4])*

I-3 The following process in which raindrops become charged is important in various meteorological effects. An initially uncharged raindrop moves into a thunder cloud which is positively charged near its top and negatively charged near its bottom (Fig. I-5). (a) What is the net charge of the top part of the raindrop and of the bottom part of the raindrop? (b) What is the charge of the entire raindrop, if no charge enters or leaves this drop as it moves into the cloud? *(Answer: 131) (Practice: [p-1])*

SECT.

J SUMMARY

DEFINITIONS

charge; Def. (A-4)

charge of a system; Def. (A-6)

electric insulator; Def. (H-1)

electric conductor; Def. (H-2)

IMPORTANT RESULTS

Coulomb's electric force law: Rule (B-4)

$\vec{F}$ is repulsive if signs of charges are the same, attractive if they are opposite; $F = k_e q_1 q_2 / R^2$ where $k_e = 9 \times 10^9\,\mathrm{N\,m^2/C^2}$.

Quantization of charge: Rule (D-1)

For any charge, $q = Ne$ where N is some integer.

Conservation of charge: Rule (E-1)

Total charge in any region remains constant if no charged particles pass between it and its surroundings.

Properties of the electron: Eq. (C-4), Eq. (C-5)

$q = -e$ where $e \approx 10^{-19}$ coulomb; $m \approx 10^{-30}$ kilogram

USEFUL KNOWLEDGE

Properties of atomic particles (Sec. C)

Charge quantization (Sec. D)

Conservation of charge (Sec. E)

Image formation by scattering in the electron microscope (Sec. G)

Electric properties of matter (Sects. H and I)

NEW CAPABILITIES

You should have acquired the ability to:

(1) Understand these relations:
  (a) The definition of charge (Sec. A)
  (b) Coulomb's electric force law (Sec. B)

(2) Qualitatively relate: the electric force on a charged particle, the motion of this particle, and the charge distribution of nearby objects (Sects. B, F, and G).

(3) For two interacting charged particles, relate: the distance between the particles, their accelerations, masses, and charges (Sects. C and G).

(4) Using a description of the initial charge on conductors or insulators to describe qualitatively the charge distribution resulting from their interaction (Sects. G and H, [p-1]).

SECT.

K PROBLEMS

Describing Electric Forces due to Several Particles

Problems K-1 and K-2 require using Coulomb's electric force law with the superposition principle to describe the force on one particle due to several others.

K-1 The particles X, 1, and 2 shown in Fig. K-1 are at the corners of an equilateral triangle. Particles 1 and 2 have charges of equal magnitude, but with the signs indicated in Fig. K-1. What is the direction of the electric force on X due to 1 and 2 (a) if X has a positive charge, and (b) if X has a negative charge? *(Answer: 122) (Suggestion: [s-13])*

K-2 Four particles with equal charges are located at the corners of a square. What is the electric force due to these particles on a negatively charged particle located at the center of the square? *(Answer: 126) (Suggestion: [p-2])*

K-3 *Estimating the nuclear force:* The nucleus of a helium atom contains two protons each with a charge of 1.6×10^{-19} coulomb and separated by a distance of about 1×10^{-15} meter. Because these protons remain in the nucleus, the magnitude of the attractive "nuclear force" between these particles must be approximately equal to the magnitude of the repulsive electric force between them. What is the magnitude of this force? Compare this magnitude with the magnitude of the gravitational force $\vec{F}_g$ on a 20 kg child due to the earth. *(Answer: 130)*

K-4 *A mass spectrometer:* A mass spectrometer, a device for determining the charge-to-mass-ratio q/m for atomic particles, can be constructed in the following way. A particle with a known initial horizontal velocity $\vec{v}_0$ enters the device through a slit located at the left in Fig. K-2. The charged plates then exert a constant upward electric force on the particle so that it follows a path like the one shown. This electric force $\vec{F}$ is proportional to the charge q of the particle, i,e., $\vec{F} = qE\hat{y}$, where E is a constant which depends on the charges of the two plates. (a) Write an expression for the charge to mass ratio q/m of a particle in terms of the distances Δx and Δy and other quantities specified in this problem. (b) Such a mass spectrometer can be used to distinguish between atomic particles. For example, suppose that a particle has a charge-to-mass ratio of 4.8×10^7 coulomb/kg. Is this particle a proton

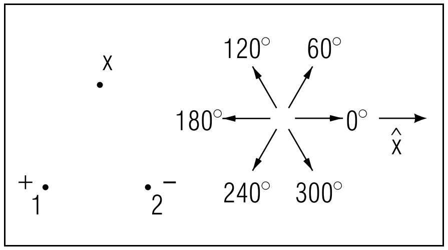


Fig. K-1.

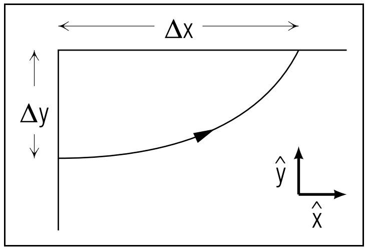


Fig. K-2.

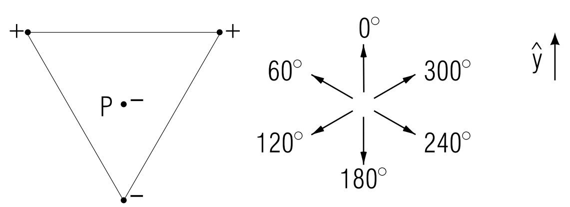
with a charge of 1.6×10^{-19} coulomb and a mass of 1.7×10^{-27} kg, or is it an alpha particle with a charge of 3.2×10^{-19} coulomb and a mass of 6.6×10^{-27} kg? *(Answer: 129) (Suggestion: [s-9])*

PRACTICE PROBLEMS

p-1 *QUALITATIVELY DESCRIBING CHARGE DISTRIBUTIONS (CAP. 4):* Cars often become charged by friction as they travel along a highway. Just before a toll collection booth, a strip of metal sticks up from the road so that it touches the metal underside of each passing car. The other end of the strip is connected by a metal rod to the earth. Suppose a car is initially negatively charged. (a) As the car drives over the metal strip, does the magnitude of its charge increase, decrease, or remain the same? (b) Suppose the metal strip is bent, and so never touches the car. Then as he pays his toll, the driver of the car extends his arm so that it touches both the metal car and the initially uncharged metal toll booth. What then happens to the charge of the car? *(Answer: 6) (Suggestion: Review your work in text problem I-2 and I-3.)*

A More Difficult Practice Problem (Text Section K)

p-2 *DESCRIBING ELECTRIC FORCES DUE TO SEVERAL PARTICLES:* Each of the particles in the following drawing has a charge with the magnitude 2×10^{-8} coulomb and having the sign indicated in the drawing.



The particle P is in the center of the equilateral triangle formed by the three other particles. What is the direction of the electric force on P due to the other particles? *(Answer: 4) (Suggestion: review text problems K-1 and K-2.)*

SUGGESTIONS

s-1 *(Text problem C-1):* In part (b), because the drop is at rest, the sum of the known gravitational force and the desired electric force must be zero.

In part (c), apply the definition of charge to relate the known forces on the two droplets to their charges. (Review text section A if necessary.)

In part (d) compare the charge -1.6×10^{-19} coulomb of the electron with the charge of the drop D in order to find the number of excess electrons on D.

s-2 *(Text problem D-2):* To find the changed charge of the sphere, you will need to add the initial charge of the sphere to the charge of the electron. It may be helpful to express this charge as -1.6×10^{-19} coulomb $= -0.000000000016 \times 10^{-8}$ coulomb.

s-3 *(Text problem G-3):* To find an expression for the electron's acceleration, apply the equation of motion $m\vec{a} = \vec{F}$ and Coulomb's electric force law to the motion of the electron.

To find the electron's speed, remember that the speed v of a particle moving with constant speed along a circular path is related to the magnitude a of its acceleration by $a = v^2/r$, where r is the radius of the circular path.

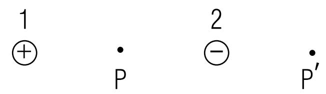
s-4 *(Text problem I-2):* If the surgeon wears shoes with conducting soles, the net charge on his body (which is a conductor) can pass through the soles into the operating room floor, thus reducing the magnitude of his charge. If the soles are insulating, such flow of charge cannot occur, and his charge must remain the same. The electric properties of graphite and rubber are discussed in text problem I-1.

s-5 *(Text problem H-1):* The symbols "+" and "−" in a drawing such as that in Fig. H-4 indicate a net excess of positively or negatively charged particles. Thus in the eel's tail region, there are many particles with both positive and negative charges, but there are a *few* more negatively charged particles (electrons) than particles with positive charge.

s-6 *(Text problem C-2):* Use the equation of motion $m\vec{a} = \vec{F}$ for the ion, using the fact that the force $\vec{F}$ on the ion is just the Coulomb electric force due to the charged sphere.

$\boxed{\textbf{s-7}}$ *(Text problem B-6):*

For each of the points P and P', draw labeled arrows representing the electric force on the positively charged particle due to the particle 1 and due to the particle 2. Draw these arrows so that an arrow of larger length indicates a force of larger magnitude.



Forces on the particle at P:

►

Forces on the particle at P':

►

(Answer: 5)

$\boxed{\textbf{s-8}}$ *(Text problem H-2):* When two conductors are touched together, the charged atomic particles can move freely and tend to spread out over the surface of both objects.

When two insulators are touched together, charged atomic particles may move from one object to the other at the point of contact, but these particles can move only slightly from their initial positions, and cannot spread out from the point of contact.

In either case, because like charges repel each other, when the balls are touched together, a few negatively charged particles will move to the initially uncharged ball. Thus the final charge distribution shown in drawing (1) is not possible.

$\boxed{\textbf{s-9}}$ *(Text problem K-4):* The particle is acted on by a constant upward force of magnitude $F = Eq$, Thus it has a constant upward acceleration of magnitude $a = Eq/m$. This situation is very similar to the motion of a particle moving subject only to gravitational interaction with the earth. The horizontal component of the particle's acceleration is zero, and the vertical component of this acceleration is constant. Review the discussion of such motion in text section E of Unit 407. You may also find it helpful to review Relation (H-4) of Unit 407.

$\boxed{\textbf{s-10}}$ *(Text problem B-3):* Let us briefly review two relations describing charge. Coulomb's electric force law described the electric force on one particle due to another particle, and therefore does not relate directly the electric forces and charges described in problem B-3.

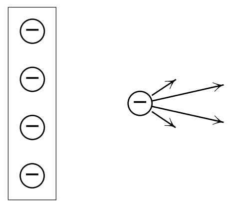
The definition of charge:

$$\frac{q_1}{q_2} = \pm\frac{F_1}{F_2}$$

relates the charges of each of two particles to the electric forces on these particles due to *another* object. The two particles must be (successively) at the same location. The object exerting the electric forces on these particles need not be a particle. In problem B-3 we can use this relation to find the force on the sulfate ion due to the root tip from the known charges of both ions and the known force on the sodium ion due to the root tip.

$\boxed{\textbf{s-11}}$ *(Text problem H-3):* If the electroscope is charged, the charged atomic particles spread out over the surface of the lower part of the rod and the metal foil. Then the foil is lifted away from the vertical by repulsive electric forces due to the rod. Therefore, when the charge of the electroscope is larger in *magnitude*, the foil is lifted farther. Whether the *sign* of the electroscope's charge is positive or negative has no effect on this process, The foil is lifted in the same way whether the interacting particles are positively or negatively charged.

$\boxed{\textbf{s-12}}$ *(Text problem B-5):* The force on a particle due to an object is the vector sum of all the individual forces on this particle due to each of the particles in the object. For example, the force on the pollutant particle due to plate 1 is the vector sum of the individual forces on the pollutant particle due to each of the excess negatively charged particles in the plate 1. Each of these individual forces is directed from the pollutant particle *away* from the particle in the negatively charged plate 1.

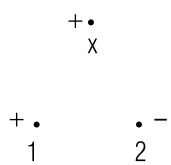


Thus the vector sum of all these individual forces is directed roughly from the pollutant particle *away* from plate 1, or towards the *right*.

s-13 *(Text problem K-1):* Since particles 1 and 2 have charges of equal magnitude, they exert on particle X forces of equal magnitude. Suppose X has a positive charge.

Use the following diagram to draw arrows (beginning at the position of X) which indicate the directions of the force $\vec{F}_1$ on X due to 1 and of the force $\vec{F}_2$ on X due to 2.

►



According to the superposition principle, the electric force on X due to 1 and 2 is simply the vector sum of the forces F_1 and F_2.

(Answer: 2)

s-14 *(Text problem F-2):* Let us consider a simpler question first. In order to deflect the beam of negatively charged electrons so as to strike the point A, the plate H_1 is negatively charged and the plate H_2 is positively charged as shown in Fig. F-1. (The plates V_1 and V_2 are uncharged.)

What are the signs of the charges on the plates H_1 and H_2 needed to deflect the electron beam so as to strike a point directly below the center?

► H_1: ____________

H_2: ____________

If the horizontal plates remain charged in this way, the beam will continue to be deflected downward.

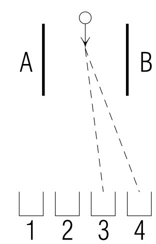
What are the signs of the charges of the plates V_1 and V_2 needed to deflect the beam towards the left, so that it strikes a point below and to the left of the center C?

► V_1: ____________

V_2: ____________

(Answer: 3)

s-15 *(Text problem F-1):* Compare the electric forces required to deflect the positively charged droplet into container 3 and into container 4.



Is the force needed to deflect the droplet into container 4 larger or smaller than that needed to deflect it into container 3?

► larger, smaller

Do these forces have the same or opposite directions?

► same, opposite

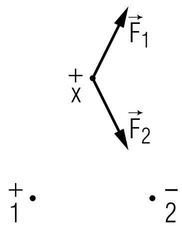
To produce an electric force on the droplet which is larger in magnitude, the magnitude of the charges on the plates must be made larger. The direction of the force exerted by the plates will remain the same if the signs of the charges on the plates remain the same.

(Answer: 1)

ANSWERS TO PROBLEMS

1. larger, same

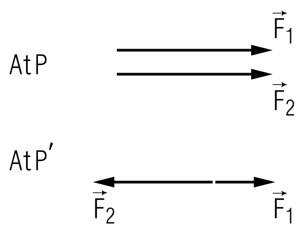
2.



3. $(H_1 : +)$ $(H_2 : -)$ $(V_1 : +)$ $(V_2 : -)$

4. Along $\hat{y}$

5.



6. a. Decreases.

   b. Again the magnitude of the car's charge decreases, but this time the charge flows through the driver's arm into the toll booth, possibly inflicting a painful shock.

101. (1) The gravitational force cannot be repulsive.

     (2) The gravitational force between two ping-pong balls is much too small to visibly affect the direction of the supporting thread.

102. The root tip is not a particle.

103. right, left

104. a. any equation equivalent to: $q_A/q_B = \pm F_A/F_B$

   b. 1×10^{-9} coulomb

   c. -1×10^{-9} coulomb

105. -3.2×10^{-19} coulomb, 4.0×10^{-17} N toward the right

106. 3.2×10^{-20} coulomb, same

107. a. 2.5×10^{-17} N left

     b. 2.5×10^{-17} N right

108. a. 8×10^{-15} N downward

     b. 8×10^{-15} N upward

     c. -3.2×10^{-19} coulomb

     d. 2

     e. $n = (2 \times 10^{-11})\,$N

109. a. number, $(+, 0, -)$, coulomb

     b. number, + or zero, kilogram

     c. 10^{-8} coulomb

110. a. $F = k_e q_1 q_2 / R^2$ (or equivalent using your choice of symbols)

     b. 0.004 N

     c. 0.02 N

     d. 0.2

111. right, right, plate 2

112. a. -1.6×10^{-9} coulomb

     b. Charged particles can pass onto the ball from the rod.

113. $q_1/q_2 = \pm F_1/F_2$ (or definition of charge), -4×10^{-23} N $\hat{x}$

114. a. $a = k_e Q q/(m R^2)$

     b. $a = 1.3 \times 10^{15}$ m/s^2

     c. away

115. Electron: (b), (c), (e). Light: (a), (d), (f). [The stains for light microscopy are the dyes methylene blue, malachite green, and crystal violet. The stains for electron microscopy are the compounds Osmium tetroxide, Lead Orthophosphate, and Phospho-tungstic acid.]

116. a. 2.0×10^{11}

b. -3.256×10^{-8} coulomb; No

117. a. 2

 b. 3

118. $(H_1 : +)$ $(H_2 : -)$ $(V_1 : +)$ $(V_2 : -)$

119. -4.8×10^{-19} coulomb. The others are not integral multiples of $e = 1.6 \times 10^{-19}$ coulomb

120. a. 3

 b. 1

 c. Cannot tell because foil is lifted in the same way whether charge is positive or negative.

121. Electron: (2). Alpha particle: (1).

122. For X positive, along $\hat{x}$. For X negative, opposite to $\hat{x}$ (or with a direction 180° from $\hat{x}$).

123. a. Container 1

 b. larger, same

124. (b) and (c) are correct.

125. Insulators: glass, rubber, distilled water. Conductors: human body, salt water, graphite.

126. Zero

127. a. $a = k_e q_e q_p/(m_e r^2)$

 b. 1×10^{23} m/s^2

 c. 2×10^6 m/s

128. a. remains the same

 b. decreases

129. a. $q/m = [2(\Delta y)v_0^2]/[E(\Delta x)^2]$

 b. It is an alpha particle.

130. 2×10^2 N. They are about the same.

131. a. Top: negative. Bottom: positive.

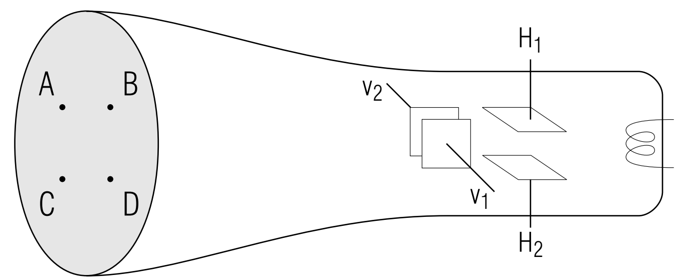
 b. Zero.

MODEL EXAM

USEFUL INFORMATION

$$k_e = 9 \times 10^9 \text{ newton meter}^2/\text{coulomb}^2$$

1. **Motion of electrons in an oscilloscope tube.** In the oscilloscope tube shown in this drawing: a beam of negatively charged electrons travels from the filament at the right towards the screen at the left. This beam is deflected by horizontal plates (H_1 and H_2) and vertical plates (V_1 and V_2) so as to trace lines on the phosphor coated screen.



 a. Suppose the plate H_1 has a charge of 2×10^{-10} coulomb. An electron, which has a charge of -1.6×10^{-19} coulomb, is located a distance of 0.5 cm from the center of H_1. Either use Coulomb's electric force law to find the electric force on the electron due to this plate, or briefly explain why this relation cannot be used to find this force from the information provided.

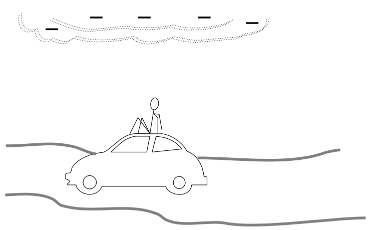
 b. At one instant during the operation of this oscilloscope tube, the four deflecting plates have charges with these signs:

 H_1: $-$ H_2: $+$

 V_1: $+$ V_2: $-$

Which of the points (A, B, C, and D) indicated in the preceding drawing best indicates the point on the screen struck by the electron beam when the deflecting plates have these charges?

2. **Charges resulting from a thunderstorm.** A foolish man watches a thunderstorm by sitting on top of his car. The man and the metal parts of the car are initially uncharged conductors in contact with each other. But because the car rests on rubber tires, as long as the air remains dry, charge cannot pass between the man and car and their surroundings. A negatively charged thundercloud passes overhead.



   a. What are the signs of the charges on the man and on the car when this cloud is overhead?

   b. If the car has a charge of magnitude 5×10^{-7} coulomb, what is the charge of the man?

3. **The charge of an ionic nucleus.** An ion consists of a nucleus of unknown positive charge and a single electron. According to a "planetary" model of this ion, the electron moves with constant speed along a circular path around the nucleus which is at rest in an inertial reference frame. Using measurements of light emitted from the ion, the electron's orbit is found to have a radius of 3×10^{-11} meter, and the electron's acceleration is found to have a magnitude of $7 \times 10^{23}\,\mathrm{m/s^2}$. The mass of an electron is 1.0×10^{-30} kg, and use the approximate value of -2×10^{-19} coulomb for the electron's charge.

   Use the equation of motion and Coulomb's electric force law to find the charge of the ionic nucleus. *Show all your work.* Your solution should be so complete and systematic that it can be understood by another person.

Brief Answers:

1. a. Coulomb's law cannot be used because: *the plate is not a particle.*

   b. D

2. a. Man: $+$, Car: $-$

   b. 5×10^{-7} coulomb

3. Evidence of using $F = k_e q_1 q_2 / R^2$ and $ma = F$.

   Must get an equation (which may include values) which is equivalent to:

   $q_1 = maR^2/(k_e q_2)$

   $m = 1.0 \times 10^{-30}$ kg

   $a = 7 \times 10^{23}\,\mathrm{m/s^2}$

   $R = 3 \times 10^{-11}$ meter

   $q_1 = 2 \times 10^{-19}$ coulomb

   Charge of ionic nucleus: (3 or 4) $\times 10^{-19}$ coulomb