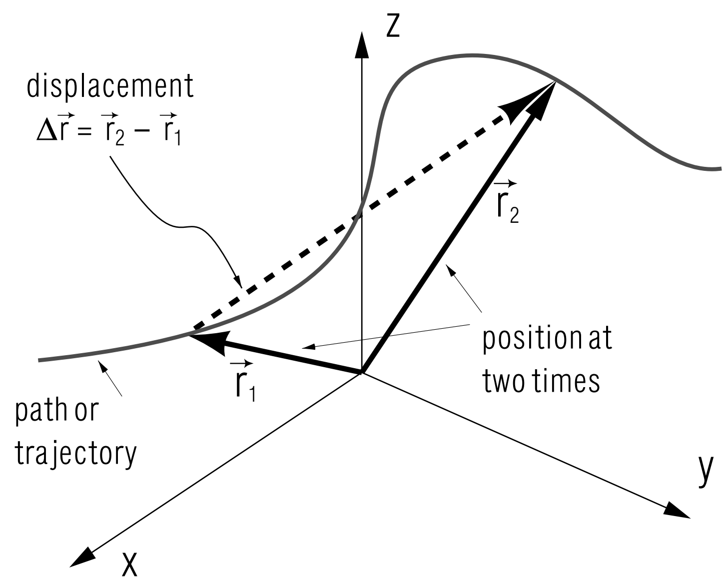
PHYSNET

MISN-0-37

KINEMATICS OF MOTION IN THREE DIMENSIONS



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

KINEMATICS OF MOTION IN THREE DIMENSIONS

by

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Title: **Kinematics of Motion in Three Dimensions**

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Version: 11/6/2001 Evaluation: Stage 0

Length: 1 hr; 24 pages

Input Skills:

1. Vocabulary: vectors, vector addition and subtraction (MISN-0-2).
2. Add and subtract vectors graphically and by components using unit vectors (MISN-0-2).

Output Skills (Knowledge):

K1. Define displacement pictorially and mathematically.

K2. Define: average and instantaneous velocity.

K3. Define: average and instantaneous acceleration.

K4. Explain the special importance of acceleration.

K5. Explain how to find the direction of the displacement, velocity and acceleration.

Output Skills (Problem Solving):

S1. Differentiate an expression that includes the unit vectors $\hat{x}$, $\hat{y}$, $\hat{z}$.

S2. Given the position of an object as a function of time, determine its velocity as a function of time, and evaluate it for a specific instant.

S3. Given the position of an object as a function of time, determine its displacement and average velocity over a given time interval.

S4. Given the velocity of an object as a function of time, determine its acceleration as a function of time, and evaluate it at a specific instant.

S5. Given the velocity of an object as a function of time, determine its average acceleration over a given time interval.

Post-Options:

1. "Kinematics in One Dimension" (MISN-0-7).
2. "Kinematics in Two Dimensions: Problem-Solving Techniques" (MISN-0-8).
3. "Particle Dynamics: The Laws of Motion" (MISN-0-14).

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1. Introduction

1a. Kinematics: Describing Motion Quantitatively. Kinematics is the study of motion and changes in motion, ignoring forces that may be causing such changes.[1] In particular, the job of kinematics is to provide a mathematical description of the various aspects of motion and also of the connections between those aspects. Thus we develop a means of precisely describing an object's position, displacement, speed, velocity, and acceleration, and we see how we can obtain any some of these quantities from others by mathematical manipulation. After mastering kinematics, we will be ready to add *forces* to the picture and plunge into the study of *dynamics*.

1b. Words Have Special Meanings in Physics. The terms "position," "displacement," "velocity" and "acceleration" are vector quantities in physics, and they will be carefully defined and described. In layman's usage, these words do not always have the specific, highly precise, meanings that physics assigns to them. The precise physics meanings, not the layman's meanings, are used throughout science and technology.

1c. Vector Quantities With Time Dependence. The motion of an object can be described by time-varying vectors whose magnitude and/or direction change as the object moves. This time dependence of the position, velocity and acceleration is expressed formally as $\vec{r}(t)$, $\vec{v}(t)$ and $\vec{a}(t)$. We will define the relationships between these quantities and relate each one of them to the motions of objects.

1d. Describing One- and Two-Dimensional Motion. The descriptors for motion in three dimensions work perfectly well for describing motion in one or two dimensions.

One-dimensional motion is the simplest to describe, being motion along a straight line.[2] An example is a car going along on a flat straight highway, perhaps with its speed increasing and decreasing as time goes on. Another example is an elevator, travelling up and down and stopping at various floors.

[1] Kinematics (kin e mat′ iks) comes from the Greek word stem "kinema" meaning "motion." The word "cinema" comes from the same stem.

[2] See "Kinematics in One Dimension" (MISN-0-7).

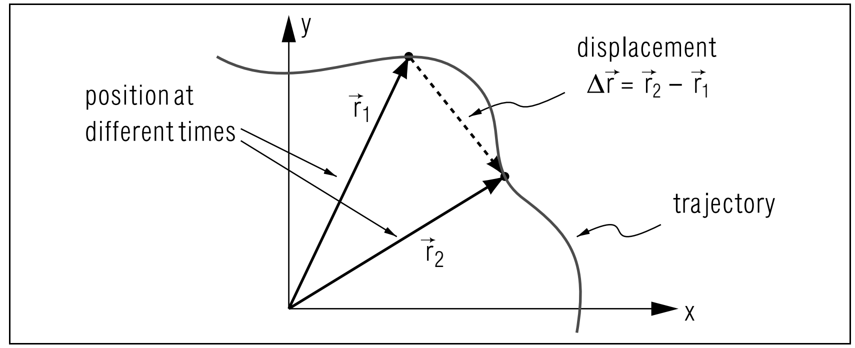


Figure 1. As an object moves, its displacement is the vector from its initial position to its current position.

Two-dimensional motion is much more complicated to describe mathematically.[3] An example is a car traveling along a winding road in a flat section of the country (to describe the position of the car one need only specify two numbers, such as the distance it is North and the distance it is East of some reference point). Another example is a car that has plunged off a cliff and is following a curving arc toward the ground, far below (while in the air the car simultaneously goes forward and down, which constitutes two dimensions).

Three-dimensional motion includes all motion and its descriptions are usually more complicated than for one- or two-dimensional motion. However, there are a number of general techniques that are the same for all three.

2. Changes of Position

2a. Displacement: The Change of the Position Vector. The displacement (vector) of a particle is defined as the change in the position vector:

$$\text{displacement} \equiv \Delta\vec{r}. \tag{1}$$

[3] See "Kinematics in Two Dimensions" (MISN-0-8).

Its magnitude and direction are given by the rules of vector subtraction.[4] Figure 1 shows the position vector for the object at two different times. By applying the definition of the displacement, you can verify that the displacement has the magnitude and direction indicated. Note that the displacement is not the distance along the trajectory.

▷ A car goes 30 miles due East and then 40 miles due North. Show that its displacement vector is: 50 miles at 37° E of N, written N37°E. *Help: [S-4]*

2b. The Average Velocity. The average velocity of an object is the displacement Δr divided by the time elapsed Δt:[5]

$$\vec{v}_{\rm av} = \frac{\Delta\vec{r}}{\Delta t}\,. \tag{2}$$

The average velocity could also be described as the average rate of change of position.

The direction of the average velocity is the same as the direction of $\Delta\vec{r}$. Since $\Delta\vec{r}$ is calculated using only initial and final values of the position over some time interval, $\vec{v}_{\rm av}$ will give no information about the velocity at any particular instant during the time interval. The instantaneous velocity, $\vec{v}$, might be small, large or varying during the interval.

Caution: At first glance, Eq. (2) might appear to be the equation you use to calculate the average speed for a trip in your car (distance traveled divided by the time interval). However, note that the displacement vector is not the distance the car travelled. In fact, if the car returns to its starting point (i.e., if $\vec{r}_2 = \vec{r}_1$), regardless of how far or how fast the car moved, its average *velocity* is zero! Of course its average *speed* would not be zero: it would be the total distance traveled, then trip mileage registered on the dash, divided by the elapsed elapsed time, the trip time.

▷ The car in the previous example spends one hour on each leg of its trip. Show that its average velocity is: 25 mi/hr, at N37°E. *Help: [S-5]*

2c. The Instantaneous Velocity. The instantaneous velocity can be envisioned as the average velocity over a very short time interval. If the time interval Δt is sufficiently small, the instantaneous velocity will not change (much) during that interval, so the average velocity will be

[4] See "Sums, Differences and Products of Vectors" (MISN-0-2).

[5] The Appendix relates this expression to the formal (calculus) definition of the average of a function.

(about) the same as the velocity at any instant in that interval.[6] This imprecise statement can be written mathematically, with precision, as:

$$\text{instantaneous velocity } = \vec{v} = \lim_{\Delta t\to 0} \vec{v}_{\rm av} = \lim_{\Delta t\to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}\,.$$

That is,

$$\vec{v} = \frac{d\vec{r}}{dt}\,.$$

2d. The Magnitude and Direction of $\vec{v}$. The instantaneous velocity is a vector having both magnitude and direction. As Δt becomes smaller, the ratio $\Delta\vec{r}/\Delta t$ will approach a finite magnitude and will approach a direction that is tangent to the trajectory. The magnitude of $\vec{v}$ is the "speed" of the particle. The direction of $\vec{v}$ is tangent to the trajectory at every instant.

▷ A car moves in such a way that the time dependence of the position vector is: *Help: [S-6]*

$$\vec{r}(t) = (1.5\,{\rm m})\,\hat{x} + (0.50\,{\rm m/s})\,t\,\hat{y} + (1.00\,{\rm m/s^2})\,t^2\,\hat{z}\,.$$

Show that its velocity at $t = 5.0$ seconds is: $(0.50\,{\rm m/s})\,\hat{y} + (10.0\,{\rm m/s})\,\hat{z}$. *Help: [S-6]*

3. Changes of Velocity

3a. A Changing Velocity Implies Acceleration. If the velocity vector of a particle is changing (in magnitude and/or direction), the particle must be accelerating. In Fig. 2, since the trajectory is curved, the velocity must certainly be changing direction.

The vector lengths show that the speed increased from t_1 to t_2. Over the time interval $\Delta t = t_2 - t_1$, the velocity has changed by an amount $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$. Vector subtraction shows the magnitude and direction of $\Delta\vec{v}$ (see Fig. 3).

3b. The Average Acceleration. The average acceleration (the average rate of change of the velocity) of an object is the change in velocity divided by the time elapsed during the change:

$$\vec{a}_{\rm av} = \frac{\Delta\vec{v}}{\Delta t}\,. \tag{3}$$

[6] If the velocity of an object is being directly measured experimentally, the position will always be measured over finite time intervals, so that the result will always be an average velocity rather than an instantaneous velocity.

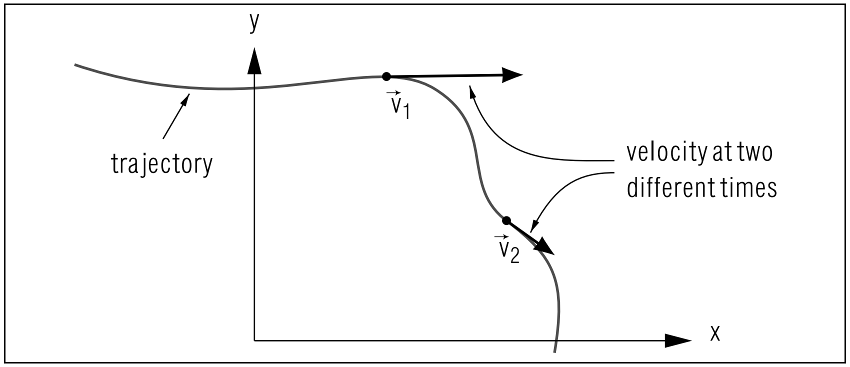


Figure 2. Velocity vectors at two times along a particle's trajectory show that the velocity has changed both its magnitude and direction.

It is a vector in the same direction as $\Delta\vec{v}$. The average acceleration gives no information about the acceleration at any particular instant during the time interval.

▷ A car started a trip travelling at 10.0 mi/hr East and ended the trip two hours later travelling at 20.0 mi/hr North. Use vector subtraction to show that its average acceleration during the trip was: 11.2 mi/hr^2 at N27°W. *Help: [S-7]*

3c. The Instantaneous Acceleration. The instantaneous acceleration can be envisioned as an average acceleration over a very short time interval. If the time interval Δt is sufficiently small, the acceleration will not change (much) during that interval, so the average acceleration will be (about) the same as the acceleration at any instant in that interval. This imprecise statement can be written mathematically, with precision, as:

$$\begin{pmatrix} \text{instantaneous} \\ \text{acceleration} \end{pmatrix} = \vec{a} = \lim_{\Delta t \to 0} \vec{a}_{\rm av} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}\,. \qquad (4)$$

The direction of the instantaneous acceleration is the same as the direction of $\Delta\vec{v}$ in the limit of $\Delta t \to 0$. Equations (3) and (4) can be combined to give:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}\,. \qquad (5)$$

▷ A car moves such that:

$$\vec{r}(t) = (1.5\,\mathrm{m})\,\hat{x} + (0.50\,\mathrm{m/s})\,t\,\hat{y} + (1.00\,\mathrm{m/s^2})\,t^2\,\hat{z}\,.$$

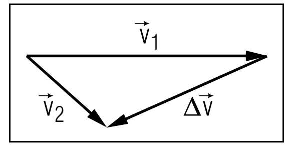


Figure 3. Change in velocity is also a vector.

Show that its acceleration at $t = 5.0$ seconds is: $2.0\,\mathrm{m/s^2}\,\hat{z}$. *Help: [S-8]*

4. Changes of Acceleration, and More

4a. What Comes Next; Jerk. So far, we have described the position and its first and second derivatives. What comes next? Higher and higher derivatives can always be defined and calculated. For example, the third derivative, the rate of change of the acceleration, is called the "jerk." It is sometimes used in engineering problems.

4b. The Importance of Acceleration. The jerk, the third derivative of position, is used far less often than is acceleration, the second derivative. The reason is that it is the acceleration of an object that is directly proportional to the total force acting on the object. Thus when we apply a known force to an object, we can immediately calculate the acceleration of the object. Then, if we wish, we can differentiate the acceleration to get the jerk. The relationship between the applied force and the resulting acceleration is developed elsewhere.[7]

Acknowledgments

Portions of this module are based upon a previous module by Prof. H. T. Hudson. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

[7] See "Particle Dynamics" (MISN-0-14).

A: the Time Average

The time average of a quantity $R(t)$ over a given interval of time $\Delta t = t_2 - t_1$, is defined as:

$$R_{\rm av} = \frac{\int_{t_1}^{t_2} R(t)\, dt}{\int_{t_1}^{t_2} dt}\,.$$

Now, if $R(t)$ is the time rate of change of a quantity $Q(t)$, that is, if

$$R = \frac{dQ}{dt}$$

then

$$R_{\rm av} = \frac{\int_{t1}^{t_2} \frac{dQ}{dt}\, dt}{\int_{t1}^{t_2} dt}\,,$$

or

$$R_{\rm av} = \frac{\int_{t_1}^{t_2} dQ}{\int_{t_1}^{t_2} dt} = \frac{Q_2 - Q_1}{t_2 - t_1} = \frac{\Delta Q}{\Delta t}\,.$$

The last expression (above) has been used throughout this module.

PROBLEM SUPPLEMENT

1. An object moves with a position that varies in time given by $\vec{r}(t) = (3.0\,\mathrm{m/s^2})\,t\,\hat{x} + (4.0\,\mathrm{m/s})\,t\,\hat{y}$. Note: If you find that you cannot work this problem, there is special assistance *Help: [S-1]*.

   a. What is its position at one second intervals from $t = 0$ to $t = 7$ seconds?

   b. Sketch its path from $t = 0$ to $t = 7$ seconds.

   c. Find $\vec{v}(t)$ for the object.

   d. Find its velocity at $t = 3.0\,\mathrm{s}$.

   e. Find its velocity at $t = 5.0\,\mathrm{s}$.

   f. Find its average velocity over the interval from 3.0 to 5.0 seconds.

   g. Find the displacement over a short time interval $\Delta t = 0.1$ second, centered around $t = 5.0\,\mathrm{s}$. Compare the average velocity over this interval to the instantaneous velocity at $t = 5.0\,\mathrm{s}$.

2. An object moves with a velocity that varies in time given by $\vec{v}(t) = (6.0\,\mathrm{m/s^2})\,t\,\hat{x} + (4.0\,\mathrm{m/s})\,\hat{y} + (3.0\,\mathrm{m/s^4})\,t\,\hat{z}$. Note: If you find that you cannot work this problem, there is special assistance *Help: [S-2]*.

   a. Find $\vec{a}(t)$ for the object.

   b. Find its acceleration at $t = 3.0\,\mathrm{s}$.

   c. Find its acceleration at $t = 5.0\,\mathrm{s}$.

   d. Find its average acceleration over the interval from 3.0 to 5.0 seconds.

3. A car sits in front of your house at $t = 0$. It then goes straight East at a constant 5.0×10^1 mph for 2.0 hours, straight North at a constant 6.0×10^1 mph for 1.0 hour, then returns along a winding road back to its original position, arriving at $t = 5.0$ hours, and is parked. Note: If you find that you cannot work this problem, there is special assistance *Help: [S-3]*.

   a. What was the displacement of the car from $t = 0$ to $t = 3.0$ hours?

   b. What was the average velocity of the car over the interval from $t = 3.0$ to $t = 5.0$ hours?

c. What was the velocity of the car at $t = 4.0$ hours?

d. What was the average velocity of the car over the interval $t = 0$ to $t = 5.0$ hours?

e. At what times did the car undergo an acceleration? (Note: Be as specific as possible, within the limitations of the wording of the question.)

4. The velocity of a particle is given by the expression $\vec{v} = (2.0\,\mathrm{m/s^2})\,t\,\hat{x} + (1.0\,\mathrm{m/s})\,\hat{y}$.

a. What is the (instantaneous) acceleration of the particle?

b. What is the average acceleration over the interval $0.0\,\mathrm{s} < t < 4.0\,\mathrm{s}$?

5. The position of an object is given by $\vec{r}(t) = (5.0\,\mathrm{m/s^3})\,t^3\,\hat{x} + (2.0\,\mathrm{m/s})\,t\,\hat{y} + (3.0\,\mathrm{m})\,\hat{z}$.

a. What is the displacement of the object over the interval $0.0\,\mathrm{s} < t < 5.0\,\mathrm{s}$?

b. What is the velocity at $t = 0.0\,\mathrm{s}$?

c. What is the velocity at $t = 5.0\,\mathrm{s}$?

d. What is the average velocity over the interval $0.0\,\mathrm{s} < t < 5.0\,\mathrm{s}$?

6. A child rides at the edge of a 5.0 m radius merry-go-round that turns once every 1.0×10^1 s. Make a sketch and indicate and calculate the average velocity of the child over the time interval $5.0\,\mathrm{s} < t < 3.0 \times 10^1\,\mathrm{s}$.

Brief Answers:

1. a. $r_0 = (0\,\mathrm{m},\ 0\,\mathrm{m})$

$r_1 = 3.0\,\mathrm{m}\,\hat{x} + 4.0\,\mathrm{m}\,\hat{y} = (3.0\,\mathrm{m},\ 4.0\,\mathrm{m})$

$r_2 = (12\,\mathrm{m},\ 8\,\mathrm{m})$

$r_3 = (27\,\mathrm{m},\ 12\,\mathrm{m})$

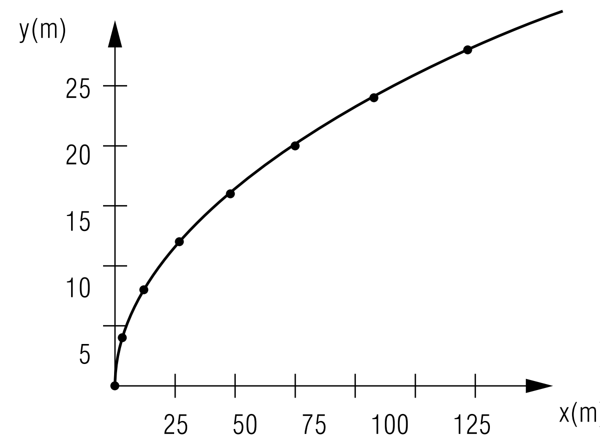
$r_4 = (48\,\mathrm{m},\ 16\,\mathrm{m})$

$r_5 = (75\,\mathrm{m},\ 20\,\mathrm{m})$

$r_6 = (108\,\mathrm{m},\ 24\,\mathrm{m})$

$r_7 = (147\,\mathrm{m},\ 28\,\mathrm{m})$.

b.



c. $\vec{v}(t) = (6.0\,\mathrm{m/s^2})\,t\,\hat{x} + (4.0\,\mathrm{m/s})\,\hat{y}$

d. $\vec{v}(3.0\,s) = (18\,\hat{x} + 4.0\,\hat{y})\,\mathrm{m/s}$

e. $\vec{v}(5.0\,s) = (30\,\hat{x} + 4.0\,\hat{y})\,\mathrm{m/s}$

f. $\vec{v}_{\mathrm{av}} = \dfrac{(75\,\hat{x} + 20\,\hat{y})\,\mathrm{m} - (27\,\hat{x} + 12\,\hat{y})\,\mathrm{m}}{5.0\,s - 3.0\,s} = (24\,\hat{x} + 4.0\,\hat{y})\,\mathrm{m/s}$

g. $\Delta\vec{r} = 3.0\,\mathrm{m}\,\hat{x} + 0.4\,\mathrm{m}\,\hat{y}$; $\vec{v}_{\mathrm{av}} = 30\,\mathrm{m/s}\,\hat{x} + 4.0\,\mathrm{m/s}\,\hat{y}$.

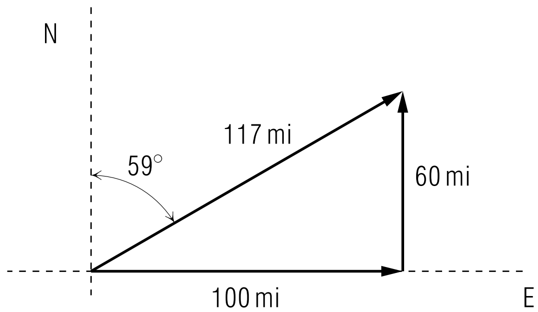
2. a. $\vec{a}(t) = (6.0\,\mathrm{m/s^2})\,\hat{x} + (3.0\,\mathrm{m/s^4})\,t^2\,\hat{z}$

b. $(6.0\,\mathrm{m/s^2})\,\hat{x} + (27\,\mathrm{m/s^2})\,\hat{z}$

c. $(6.0\,\mathrm{m/s^2})\,\hat{x} + (75\,\mathrm{m/s^2})\,\hat{z}$

d. $\vec{a}_{\mathrm{av}} = (6.0\,\mathrm{m/s^2})\,\hat{x} + (49\,\mathrm{m/s^2})\,\hat{z}$

3. a.



   b. 58.5 mi/hr at S59°W.

   c. Not enough information supplied about the return trip.

   d. 0

   e. The car must have accelerated for a short time at $t = 0$, again at $t = 2.0\,\text{hr}$, at $t = 3.0\,\text{hr}$, and at $t = 5.0\,\text{hr}$. Not enough information is given for the interval $t = 3.0\,\text{hr}$ to $t = 5.0\,\text{hr}$ to quantitatively specify the acceleration at any time in that interval. The winding road implies almost continual acceleration.

4. a. $\vec{a} = (2.0\,\text{m/s}^2)\,\hat{x}$

   b. $\vec{a}_{\text{av}} = (2.0\,\text{m/s}^2)\,\hat{x}$

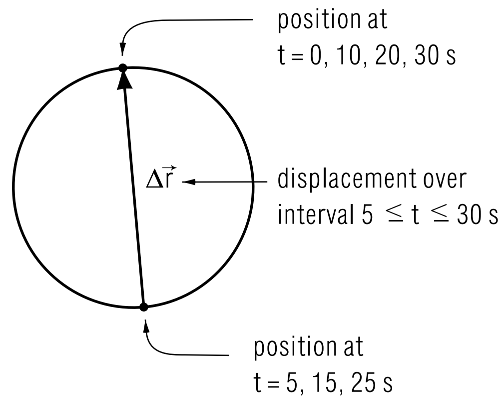
5. a. $625\,\text{m}\,\hat{x} + 1.0 \times 10^1\,\text{m}\,\hat{y}$

   b. $(2.0\,\text{m/s})\,\hat{y}$

   c. $(375\,\text{m/s})\,\hat{x} + (2.0\,\text{m/s})\,\hat{y}$

   d. $(125\,\text{m/s})\,\hat{x} + (2.0\,\text{m/s})\,\hat{y}$

6.



$$\begin{aligned}\vec{v}_{\text{av}} &= (\Delta\vec{r})/(\Delta t)\\ &= (10\,\text{m})/(25\,\text{s}),\ \text{in dir. of } \Delta\vec{r}\\ &= 0.40\,\text{m/s},\ \text{in dir. of } \Delta\vec{r}.\end{aligned}$$

SPECIAL ASSISTANCE SUPPLEMENT

S-1 *(from PS-problem 1)*

a. Example:

$$\begin{aligned}\vec{r}(2\,\mathrm{s}) &= (3)(2\,\mathrm{s})^2\,\mathrm{m/s^2}\,\hat{x} + 4(2\,\mathrm{s})\,\mathrm{m/s}\,\hat{y}\\ &= 12\,\mathrm{m}\,\hat{x} + 8\,\mathrm{m}\,\hat{y}\\ &= (12\,\mathrm{m}, 8\,\mathrm{m}).\end{aligned}$$

b. Plot the coordinates from part (a) as data on an x-y graph. You might prefer to use different scales on the two axes.

c. $\vec{v}(t) = d\vec{r}/dt = (d/dt)\left[(3\,\mathrm{m/s^2})\,t^2\,\hat{x} + (4\,\mathrm{m/s})\,t\,\hat{y}\right]$.
The units and unit vectors are constant in time so:
$\vec{v}(t) = (3\,\mathrm{m/s^2}\,\hat{x})\,(d/dt)\,t^2 + (4\,\mathrm{m/s}\,\hat{y})\,(d/dt)\,t$.

d. Evaluate $\vec{v}(t)$ at $t = 3\,\mathrm{s}$.

e. Evaluate $\vec{v}(t)$ at $t = 5\,\mathrm{s}$.

f. Use the definition of $\vec{v}_{\mathrm{av}}$, Eq. (5).
$\Delta\vec{r} = \vec{r}(5\,\mathrm{s}) - \vec{r}(3\,\mathrm{s})$; $\Delta t = 5\,\mathrm{s} - 3\,\mathrm{s} = 2\,\mathrm{s}$.
Caution: Note that $\vec{v}_{\mathrm{av}}$ is *not* defined as the average of the initial and final velocities. Rather, it is the velocity averaged over the entire time interval.

g. The time interval is $4.95\,\mathrm{s} \le t \le 5.05\,\mathrm{s}$. Evaluate $\vec{r}(t)$ at these two times and subtract to get the displacement vector. Divide $\Delta\vec{r}$ by Δt to get $\vec{v}_{\mathrm{av}}$.

S-2 *(from PS-problem 2)*

a. $\vec{a}(t) = \dfrac{d}{dt}\vec{v}(t)$. See [S-6] for hints for differentiating vectors.

b. Evaluate $\vec{a}(t)$ at $t = 3\,\mathrm{s}$.

c. Evaluate $\vec{a}(t)$ at $t = 5\,\mathrm{s}$.

d. Use the definition of $\vec{a}_{\mathrm{av}}$, Equation (7).

S-3 *(from PS-problem 3)*

a. You must first define a coordinate system for this problem. An easy choice has x East and y North, with the origin at the spot "in front of your house." That is, the initial position is $\vec{r}_1 = 0$. Then $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = \vec{r}_2$. What is the vector position $\vec{r}_2$ at $t = 3$ hours?

b. The average velocity is the displacement divided by the time. Over the interval from 3.0 to 5.0 hours, the car returns, so that its displacement is exactly equal and opposite its $0.0 - 3.0$ hour displacement.

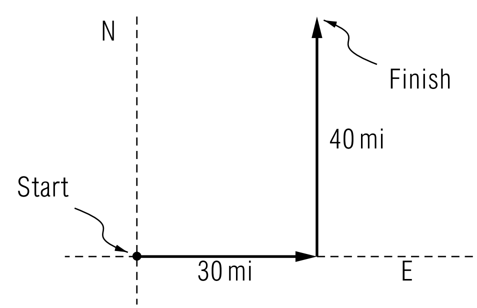
c. During the 3.0 to 5.0 hour interval, the car could slow, turn, stop, speed, etc. The average velocity over this time interval gives no information about the velocity at some instant in the interval.

d. For this time interval, the displacement vector is zero.

e. The car accelerates whenever its velocity changes magnitude and/or direction. At $t = 0.0$, it must spend a short (unspecified) time accelerating to 50 mph. At $t = 2.0$ hr, it changes both the magnitude and direction of its velocity. What does the winding road imply about velocity changes?

S-4 *(from TX-2a)*

The path of the car is shown in the sketch. You draw in the displacement vector: it begins at "Start" and ends at "Finish."



S-5 *(from TX-2b))*

The average velocity is the displacement given in [S-4] divided by the time (a total of two hours).

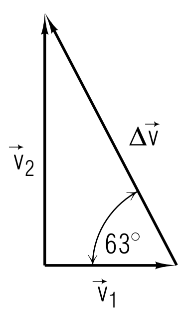
S-6 *(from TX-2d)*

$\vec{v}(t) = (d/dt)\,\vec{r} = (d/dt)\,\left[(1.5\,\mathrm{m})\,\hat{x} + (0.50\,\mathrm{m/s})\,t\,\hat{y} + (1.00\,\mathrm{m/s^2})\,t^2\,\hat{z}\right]$.
The unit vectors $\hat{x}$, $\hat{y}$, and $\hat{z}$ are constant in time, in both magnitude and direction. That is, $d\hat{x}/dt = d\hat{y}/dt = d\hat{z}/dt = 0$.
Thus:
$d/dt\,(1.5\,\mathrm{m})\,\hat{x} = 0$,
$d/dt\,(2.0\,\mathrm{m/s})\,t\,\hat{y}) = 2.0\,\mathrm{m/s}\,\hat{y}$, and
$d/dt\,(1.00\,\mathrm{m/s^2})\,t^2\,\hat{z} = (2.00\,\mathrm{m/s^2})\,t\,\hat{z}$.

S-7 *(from TX-3b)*

$$\begin{aligned}\vec{a}_{\mathrm{av}} &= \frac{\Delta\vec{v}}{\Delta t}\\ &= \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}\\ &= \frac{22.4\,\mathrm{mi/hr}}{2\,\mathrm{hr}}\\ &= 11.2\,\mathrm{mi/hr^2}.\end{aligned}$$

The direction is found from the figure:



S-8 *(from from TX-3c)*

$$\begin{aligned}\vec{a} &= \frac{d\vec{v}}{dt}\\ \vec{v} &= \frac{d}{dt}\left[(1.5\,\mathrm{m})\,\hat{x} + (0.50\,\mathrm{m/s})\,t\,\hat{y} + (1.00\,\mathrm{m/s^2})\,t^2\,\hat{z}\right]\\ &= (0.50\,\mathrm{m/s})\,\hat{y} + (2.00\,\mathrm{m/s^2})\,t\,\hat{z}\\ \vec{a} &= \frac{d\vec{v}}{dt} = (2.0\,\mathrm{m/s^2})\,\hat{z}.\end{aligned}$$

MODEL EXAM

1. See Output Skills K1-K5 in this modules's *ID Sheet*. The exam may have one or more of these skills, or none.

2. The velocity of a particle is given by the expression $\vec{v} = (2.0\,\mathrm{m/s^2})\,t\,\hat{x} + (1.0\,\mathrm{m/s})\,\hat{y}$.

   a. What is the (instantaneous) acceleration of the particle?

   b. What is the average acceleration over the interval $0.0\,\mathrm{s} < t < 4.0\,\mathrm{s}$?

3. The position of an object is given by $\vec{r}(t) = (5.0\,\mathrm{m/s^3})\,t^3\,\hat{x} + (2.0\,\mathrm{m/s})\,t\,\hat{y} + (3.0\,\mathrm{m})\,\hat{z}$.

   a. What is the displacement of the object over the interval $0.0\,\mathrm{s} < t < 5.0\,\mathrm{s}$?

   b. What is the velocity at $t = 0.0\,\mathrm{s}$?

   c. What is the velocity at $t = 5.0\,\mathrm{s}$?

   d. What is the average velocity over the interval $0.0\,\mathrm{s} < t < 5.0\,\mathrm{s}$?

4. A child rides at the edge of a 5.0 m radius merry-go-round that turns once every 1.0×10^1 s. Make a sketch and indicate and calculate the average velocity of the child over the time interval $5.0\,\mathrm{s} < t < 3.0 \times 10^1\,\mathrm{s}$.

Brief Answers:

1. See this module's text.

2. See this module's *Problem Supplement*, problem 4.

3. See this module's *Problem Supplement*, problem 5.

4. See this module's *Problem Supplement*, problem 6.