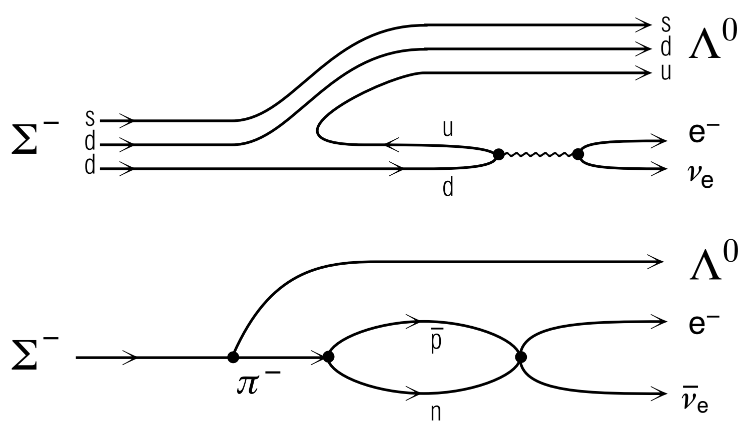
MISN-0-278

ISOSPIN: CONSERVED IN STRONG INTERACTIONS



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ISOSPIN: CONSERVED IN STRONG INTERACTIONS

by

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Title: **Isospin: Conserved in Strong Interactions**

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Input Skills:

1. State the algebraic rules for spin (MISN-0-251).
2. Calculate the hypercharge and strangeness of a particle (MISN-0-277).

Output Skills (Knowledge):

K1. Identify the hadron multiplets and list their members.

K2. Give the isotopic spin for each hadron multiplet and explain how it is related to the number of members in the multiplet.

K3. Relate the isospin concept to the hypothesis that members of a multiplet are different states of the same basic object.

K4. Enumerate the allowed values for T_3, for a given T.

K5. Give the assigned values of T and T_3 for each hadron and its antiparticle.

K6. State the relationship between charge, strangeness, baryon number and T_3.

Output Skills (Problem Solving):

S1. For a given system of particles, calculate the allowed values of the total isotopic spin.

S2. Given a decay or interaction, use the conservation laws to determine if it can occur and if so, which force is responsible.

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1. Abstract

The quantity isospin is conserved only by the strong interaction. Isospin values are assigned only to hadrons: if leptons or photons are involved then the interaction is not strong and isospin is irrelevant. The algebra of isospin is precisely the same as the algebra of angular momentum.

2. Readings

1. Sect. 27.10 in K. W. Ford's *Classical and Modern Physics*, Vol. 3, John Wiley and Sons, NY(1972).
2. Chapter 1 in M. J. Longo's *Fundamentals of Elementary Particle Physics*.

3. Isospin Concept

3a. Multiplet Splitting and Charge. Examples of Hadron multiplets are: a doublet of nucleons and a triplet of pions. The members of a multiplet have the same strangeness, hypercharge, spin, baryon number, electron family number, and muon family number, but differ in charge and differ slightly in mass (about 5 MeV differences in mass). It is believed that if electromagnetic interactions were turned off, the members of a multiplet would be identical.

This hypothesis is born out by the study of nucleon-nucleon scattering where neutron-neutron scattering can be deduced from proton-proton scattering by "turning off the proton's charge" in the scattering equations.

3b. Replace Charge by Isospin. It is convenient to define a new quantity, called isotopic spin or isospin, that accounts for the charges of the various members of a multiplet. Isospin is a little more complicated than it need be just to account for charge: it turns out that isospin is conserved in strong interactions and is more restrictive than simple charge

conservation.

3c. Reason for "Spin" in Isospin. The algebra of isospin is exactly the same as for spin, which accounts for its name. It should be emphasized at the outset, however, that isospin is not spin. Spin exists in real space but isospin exists only in an abstract mathematical isospin space.

4. Isospin Calculations

4a. Magnitude, Components. Each multiplet is assigned a isospin number T (or, more usually, I) that is a positive integer or half an odd positive integer. Isospin may be considered to be a vector with magnitude $\sqrt{T(T+1)}$ (compare with the spin vector of magnitude $\hbar\sqrt{s(s+1)}$). To emphasize that isospin is not a vector in coordinate space, we will not call the axes x, y, z but rather 1, 2, 3. The 3 component, T_3, may take on any one of the values T, $T-1$, $T-2$, …, $-T$ in a fashion similar to the values of the z component of angular momentum. Each of these values corresponds to a different member of the multiplet with $T_3 = T$ assigned to the particle with the most positive charge and $T_3 = -T$ assigned to the particle with the most negative charge. If a particle is in a state with definite T_3 , the uncertainty principle prohibits it from having definite values of T_1 or T_2 so we ignore those components.

4b. Multiplicity. If the isospin of a multiplet is T, there are $2T + 1$ particles in the multiplet. This result follows from counting the possible values of T_3. Thus singlets have $T = 0$, doublets have $T = 1/2$, and triplets have $T = 1$. Later we shall run into a quartet ($T = 3/2$).

4c. Isospin Assignments. A summary of isospin assignments is given in the chart in the Appendix. Note that an antiparticle has the same value of T as the corresponding particle but its T_3 has the opposite sign.

4d. Isospin Addition. Total isospin for a collection of particles is computed in the same manner as for ordinary spin. The magnitude of the total isospin is $\sqrt{T(T+1)}$ where T is a positive integer or half an odd positive integer and, for a given collection of particles, the maximum value T can have is the sum of the individual particle's T's:

$$T_{\max} = \sum_i T(i).$$

The value of T_3 for a collection of particles is the sum of the individual particle's T_3's:

$$T_3 = \sum_i T_3(i).$$

and T must be greater than or equal to $|T_3|$. Thus T can have any one of the values $T_{\max}$, $T_{\max-1}$, $T_{\max-2}$, ..., $|T_3|$.

For example if one considers π^+-p scattering, $T_{\max} = 3/2$ and $T_3 = 3/2$ so T can only have the value 3/2. For $\pi^- - p$ scattering, $T_{\max} = 3/2$ and $T_3 = -1/2$ so T can be either 3/2 or 1/2. In fact, measurement of T for π^--p sometimes produces $T = 3/2$ and sometimes $T = 1/2$ but always $T_3 = -1/2$.

5. Isospin Conservation

5a. Relationship: Q, S, B, T_3; Y, T_3. Charge, baryon number, and the 3 component of isotopic spin are related. The charge of a particle is given by:

$$q = \frac{S+B}{2} + T_3 = \frac{Y}{2} + T_3.$$

The first term, $(S+B)/2$, gives the average charge of the multiplet (or half the hypercharge) and is the same number for all particles in the multiplet. T_3 measures additional charge the specific particle has. Note that the signs of S, B, T_3, and q are opposite for a particle and its antiparticle.

5b. Conservation of T, T_3; Strong, EM, Weak. Both T and T_3 are conserved during strong interactions and decays. Since q, S, and B are conserved there, the conservation of T_3 is not a new conservation law (see equation of Sect. 3a). For weak interactions neither S, T_3, nor T need be conserved although q and B are universally conserved. For electromagnetic interactions, T need not be conserved.

6. Applications

6a. Experimental Consequences. The conservation of isospin prohibits some decays from taking place via the strong interaction.

For example (see the chart of Sect. 2), consider the decay of the Σ^0. The products must include a single baryon (conservation of energy prohibits 2 baryons and an antibaryon). If the decay is to be strong the product particles must have strangeness -1. This requirement leads to

the conclusion that one of the products is either Λ^0 or one of the antikaons. A kaon and any baryon together are too massive so the decay must be to a Λ^0. Any other particles that could be produced must have total mass less than 76.9 MeV: i.e., photons, electrons, or neutrinos.

We have determined that the decay products of Σ^0 must be the Λ^0 plus photons or leptons. Can this decay take place via the strong interaction? The Σ^0 has isotopic spin 1 while the Λ^0 has isotopic spin 0. All the other possible products (leptons and photons) have isotopic spin 0. We conclude that the decay cannot take place via the strong interaction. However all the conservation laws for the electromagnetic interaction do hold—in fact the decay $\Sigma^0 \to \Lambda^0 + \gamma$ does not violate any conservation law we have discussed except isospin. The decay takes place via the electromagnetic interaction.

6b. Deducing Interactions from Decay Products. You are now in a position to figure out which of the three forces is responsible for a decay or interaction for which the products are given. Assume that the interaction proceeds via the fastest route for which the appropriate quantities are conserved.

A few notes:

i. If particles appear for which isospin is not defined (γ, leptons), then the interaction is not strong.

ii. If neutrinos appear, then the interaction must be weak.

iii. If photons appear, the interaction must be electromagnetic.

iv. It is sometimes impossible to distinguish between weak and electromagnetic interactions on the basis of product particles alone. In particular, when C, P, or T violation occurs, the products may not show the violation (neutrinos, of course, do).

v. What you are really doing is finding the dominant part of the interaction for the determination of lifetime. We shall see in the next section that some interactions and decays proceed by two or more steps. Nevertheless it is worthwhile to practice predicting the type of decay from conservation law restrictions.

6c. Examples.

a. $\pi^- + \mathrm{p} \to \mathrm{K}^0 + \Lambda^0$

This interaction conserves charge and baryon number. It can conserve angular momentum, energy, and momentum. It conserves strangeness. For the pion, $T = 1$, $T_3 = -1$; for the proton, $T = 1/2$, $T_3 = 1/2$; for the kaon, $T = 1/2$, $T_3 = -1/2$; and for the lambda $T = 0$, $T_3 = 0$. Initial states could be $T = 3/2$, $T_3 = -1/2$ or $T = 1/2$. The final state must be $T = 1/2$, $T_3 = -1/2$. We conclude that isotopic spin could be conserved. So, barring violation of C, P, or T, the interaction is strong.

b. $\mathrm{p} + \mathrm{p} \rightarrow \mathrm{K}^0 + \mathrm{p} + \mathrm{p}$

This interaction does not conserve strangeness so it must be weak. Charge and baryon number are conserved and angular momentum, energy, and momentum could be conserved.

6d. Predicting Products of Decays or Interactions. In some cases you can use the conservation laws to predict the outcome of a decay or interaction. List all possible outcomes that conserve the universally conserved quantities (to use the conservation of energy law, you will need to know or assume the kinetic energy of the incident particle). Search for a combination that does not violate any conservation laws for the strong interaction. If a combination is found, that will in fact be the outcome. If two or more are found they will generally compete and all of them will be seen experimentally. If none are found proceed to the electromagnetic interaction and repeat the process. If none are found, finally consider the weak interaction.

It is important to proceed in the order: strong, electromagnetic, weak. If an interaction can go via either strong or electromagnetic, it will go via the strong since this is so much faster then the electromagnetic, etc.

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A. Table of Isospin Values

Multiplet	T	Members	T_3
Pion	1	π^+	+1
		π^0	0
		π^-	−1
Kaon	1/2	K^+	+1/2
		K^0	−1/2
Eta	0	η	0
Nucleon	1/2	p	+1/2
		n	−1/2
Lambda	0	Λ^0	0
Sigma	1	Σ^+	+1
		Σ^0	0
		Σ^-	−1
Xi	1/2	Ξ^0	+1/2
		Ξ^-	−1/2
Omega	0	Ω^-	0

PROBLEM SUPPLEMENT

1. Give possible values for the total isotopic spin T and for its 3-component T_3 for each of these systems of particles:

a. π^0 - p	f. Σ^0 - p
b. π^+ - p	g. Λ^0 - n
c. π^- - n	h. K^- - K^+
d. π^0 - n	i. K^0 - Ξ^-
e. p- n	j. Λ^0 - Ω^-

2. Figure out which force is responsible for each of the following decays or interactions. Justify your answer. If the decay or interaction does not occur, so state and name the conservation law that is violated.

a. $\mathrm{n} \to \mathrm{p} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$	d. $\overline{\mathrm{p}} + \mathrm{p} \to \mathrm{e}^+ + \mathrm{e}^-$
b. $\pi^- + \mathrm{p} \to \pi^0 + \mathrm{n}$	e. $\mathrm{p} + \mathrm{p} \to \pi^+ + \mathrm{d}$
c. $\Omega^- \to \mathrm{K}^- + \overline{\mathrm{K}}^0$	f. $\mathrm{n} + \mathrm{p} \to \eta + \mathrm{d}$

Here d = deuteron = (n + p). Experimentally it is found that $T = 0$ for the deuteron.

3. The Δ^{++} is a particle with mass 1236 MeV, charge $2\,|e|$, spin $\hbar/2$, isospin $T = T_3 = 3/2$, strangeness 0, and baryon number 1. What are the decay products of the Δ? What force is responsible for the decay? Consider only particles stable against strong decay.

4. Suppose the Δ has all the properties listed in problem 3 except that its mass is 1050 MeV. What would then be its decay products? What force would be responsible for the decay? Consider only particles stable against strong decay.