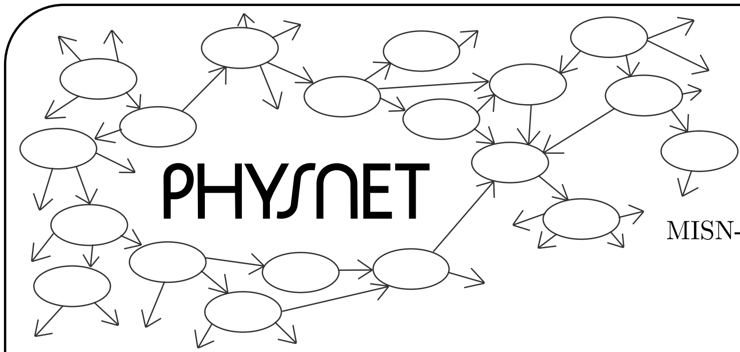
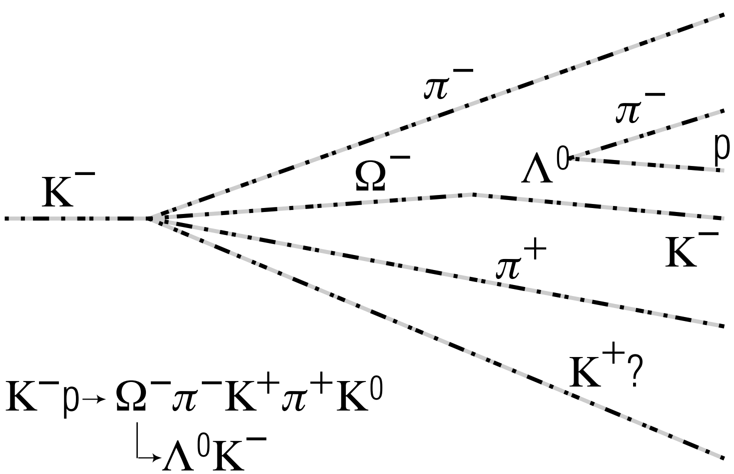


MISN-0-24

RELATIVISTIC MOMENTUM: PARTICLE DECAYS



$$K^- p \to \Omega^- \pi^- K^+ \pi^+ K^0$$
$$\hookrightarrow \Lambda^0 K^-$$
$$\hookrightarrow p \pi^-$$

Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

RELATIVISTIC MOMENTUM: PARTICLE DECAYS

by
Peter Signell

Title: **Relativistic Momentum: Particle Decays**

Author: P. Signell, Dept. of Physics, Mich. State Univ

Version: 2/1/2000 Evaluation: Stage B1

Length: 1 hr; 12 pages

Input Skills:

1. Work simple conservation of momentum problems (MISN-0-14).
2. Use the relativistic form for total energy in conservation of energy problems (MISN-0-23).
3. Expand the square root function in a Taylor series about a given point (MISN-0-4).

Output Skills (Knowledge):

K1. Reduce the expression for relativistic momentum to its non-relativistic form, using the general expression for Taylor's Series for the expansion of a function about a point.

K2. Show that $\vec{F} = m\vec{a}$ is generally valid only for $v^2 \ll c^2$.

Output Skills (Problem Solving):

S1. Given a particle's rest mass and velocity, calculate its relativistic momentum and energy.

S2. Use conservation of energy and momentum to work decay problems, 1 particle $\Rightarrow$ 2 particles, in the center of mass frame.

External Resources (Required):

1. M. Alonso and E. J. Finn, *Physics*, Addison-Wesley (1970), or R. Resnick, *Basic Concepts in Relativity and Early Quantum Theory*, John Wiley & Sons (1972), or R. T. Weidner and R. L. Sells, *Elementary Modern Physics*, Allyn and Bacon (1980). For availability, see this module's *Local Guide*.

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Our publications are designed: (i) to be updated quickly in response to field tests and new scientific developments; (ii) to be used in both classroom and professional settings; (iii) to show the prerequisite dependencies existing among the various chunks of physics knowledge and skill, as a guide both to mental organization and to use of the materials; and (iv) to be adapted quickly to specific user needs ranging from single-skill instruction to complete custom textbooks.

New authors, reviewers and field testers are welcome.

Views expressed in a module are those of the module author(s) and are not necessarily those of other project participants.

RELATIVISTIC MOMENTUM: PARTICLE DECAYS

by

Peter Signell

1. Readings Alternatives: Choose One

1a. Alternative 1: AF. Read in AF:[1] Sections 12.4, 12.5, and 12.6. Optional Chapter 12 problems: 2, 15, 22, 28.

1b. Brief Answers.

2. When $v = \dfrac{1}{\sqrt{2}}c$

   $E_{\text{total}} = \sqrt{2}m_0c^2$

   $E_{\text{kin}} = (\sqrt{2} - 1)m_0c^2$

15. Electron: 0.788 MeV, 0.761 c

    Proton: 938.3 MeV, 1.92×10^5 m/s $= 6.39 \times 10^{-4}$ c

22. $\vec{p}_1\prime = \vec{p}$

    $\vec{p}_2\prime = -\vec{p}$

    where

    $$p = \frac{c}{2m_0}\left(m_0^4 + m_1^4 + m_2^4 - 2m_0^2m_1^2 - 2m_0^2m_2^2\right)^{1/2}$$

1c. Alternative 2: RR. Read in RR:[2] Sections 3.2, 3.3, and 3.4 with the exception of those areas in 3.4 pertaining to electric and magnetic fields. Optional Problems: 20, 24, and 29 on pp. 106-109.

[1] M. Alonso and E. J. Finn, *Physics*, Addison-Wesley (1970) (see this module's Local Guide for details on obtaining this reference).

[2] R. Resnick, *Basic Concepts in Relativity and Early Quantum Theory*, Wiley(1972) (see this module's Local Guide for details on obtaining this reference).

1d. Brief Answers.

20. 4.42×10^{-36} kg; 2.208×10^{-32} kg.

24. b. 0.511 MeV

    c. 938 MeV.

29. a. $(7/12)\,\mathrm{M_0c}$

    b. $(1/5)\,\mathrm{M_0c}$

    c. $(32/12)\,\mathrm{M_0c^2}$

    d. $(34.29/12)\,\mathrm{M_0}$

    e. $(0.71/12)\,\mathrm{M_0c^2}$

1e. Alternative 3: WSM. Read in WSM:[3] Sections 3-1, 3-2, and 3-3, including the examples. For the purposes of this module it is not necessary to learn the derivations given in Section 3-1; only the results.

Optional Chapter 3 problems: 1, 8, 25, pp. 83-4.

1f. Brief Answers.

1. 0.87 c

8. 8.3×10^4

25. 5.6×10^{19} J for spaceship alone

2. Work These Problems

2a. Problem A. The rest mass of a π^+ ("pi-plus") meson is about 140 MeV/c^2. If a π^+ is traveling at 0.8 c, compute its energy in MeV and momentum in MeV/c.

[3] R. Weidner and R. Sells, *Elementary Modern Physics*, 3rd Edition, Allyn and Bacon (1980) (see this module's Local Guide for details on obtaining this reference).

2b. Problem B. In the decay of a Λ^0 ("lambda-zero") particle, at rest, to a neutron (n) and a π^0 ("pi-zero"), use conservation of energy and momentum to show that the momentum of the neutron is 104 MeV/c.

$$\begin{aligned} \text{mass of } \Lambda^0 &= 1115.63\,\mathrm{MeV/c^2} \\ \text{mass of n} &= 939.5656\,\mathrm{MeV/c^2} \\ \text{mass of } \pi^0 &= 134.9739\,\mathrm{MeV/c^2} \end{aligned}$$

Note: $(\text{momentum})^2 = k^2v^2m_0^2 = (k^2-1)m_0^2c^2$ is a useful identity that you can easily prove.

Note: For further decay problems see, "Review of Particle Properties," *Physics Letters*, Vol. 239, April 12, 1990.

You might also like to try Problem 12.28, AF.

Acknowledgments

We wish to thank Ben Oh, J. Whitmore and G. A. Smith of Michigan State University for the cover photograph showing two 2-body decays as well as a 5-body one. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

Answers to Problems

A. 233 MeV, 187 MeV/c.

B. Consv. of E: $m_\Lambda = k_\mathrm{n}m_\mathrm{n} + k_\pi m_\pi$

Consv. of $\vec{p}$: $0 = k_\mathrm{n}\vec{v}_\mathrm{n}m_\mathrm{n} + k_\pi\vec{v}_\pi m_\pi$

Squaring the second equation and using the identity in the Note,

(a) $(k_\mathrm{n}^2-1)m_\mathrm{n}^2 = (k_\pi^2-1)m_\pi^2$.

Squaring the first equation after solving it for $k_\pi m_\pi$:

(b) $k_\pi^2m_\pi^2 = m_\Lambda^2 - 2m_\Lambda k_\mathrm{n}m_\mathrm{n} + k_\mathrm{n}^2m_\mathrm{n}^2$.

Then putting (b) into (a) to eliminate k_π gives:

$$k_\mathrm{n} = \frac{m_\Lambda^2 - m_\pi^2 + m_\mathrm{n}^2}{2m_\Lambda m_\mathrm{n}}\,.$$

Again using the identity we find:

$$(\text{momentum})_\mathrm{n} = \left(k_\mathrm{n}^2-1\right)^{1/2} m_\mathrm{n}c = 103.9\,\mathrm{MeV/c}\,.$$

LOCAL GUIDE

The readings for this unit are on reserve for you in the Physics-Astronomy Library, Room 230 in the Physics-Astronomy Building. Ask for them as "The readings for CBI Unit 24." Do **not** ask for them by book title.

PROBLEM SUPPLEMENT

Note: Problems 1, 3, and 4 also occur on this module's *Model Exam.*

1. The rest mass of a Σ^+ ("sigma-plus") particle is $1189.37\,\mathrm{MeV/c^2}$. If it is traveling at speed $0.8000\,\mathrm{c}$, compute its energy in MeV and its momentum in MeV/c.

2. Reduce the expression for relativistic momentum to its non-relativistic form, using the general expression for Taylor's Series for the expansion of a function about a point.

3. In the decay of a Σ^+ particle at rest, to a proton and a π^0 particle, calculate the momentum of the proton in MeV/c.

$$\begin{aligned} m_{\mathrm{p}} &= 938.2592\,\mathrm{MeV/c^2} \\ m_{\pi^0} &= 134.9645\,\mathrm{MeV/c^2} \\ m_{\Sigma^+} &= 1189.37\,\mathrm{MeV/c^2} \end{aligned}$$

   Compare your answer to the (189 MeV/c) listed in "Review of Particle Properties," *Physics Letters*, Vol. 239, April 12, 1990.

4. Show that $\vec{F} = m\vec{a}$ is generally valid only for $v^2 \ll c^2$.

Brief Answers:

1. 1982 MeV, 1586 MeV/c.
2. Taylor's Series:
   $f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f}{2!}x^2 + \ldots$
   $k(x) = (1-x)^{-1/2}$ where $x \equiv v^2/c^2$ for our case
   $k'(x) = (1/2)(1-x)^{-3/2}$
   $k''(x) = (3/4)(1-x)^{-5/2}$
   $k(x) = 1 + (1/2)x + (3/8)x^2 + \ldots$
   $k(v^2) = 1 + (1/2)(v^2/c^2) + (3/8)(v^4/c^4) + \ldots$
   mom. $= kvm_0 = m_0v + (1/2)m_0(v^3/c^2) + \ldots$
   Then if $v^2 \ll c^2$ we can neglect the second term and get:
   $$\text{mom.} = m_0 v, \qquad \text{for } v^2 \ll c^2.$$
3. As in Problem B, derive:
   $$k_p = \frac{m_\Sigma^2 - m_\pi^2 + m_\mathrm{p}^2}{2m_\Sigma m_b p}$$
   and find the numerical value of the momentum.
4. Newton's Second Law, verified relativistically, is:
   $$\vec{F} = \frac{d\vec{p}}{dt},$$
   but then:
   $$\begin{aligned}\vec{F} &= \frac{d(m\vec{v})}{dt}\\ &= m\vec{a} + \vec{v}\frac{dm}{dt}, \qquad \text{where } m \equiv km_0\\ &\neq m\vec{a},\ \text{unless } v^2 \ll c^2 \text{ so } dm/dt = 0\,.\end{aligned}$$

MODEL EXAM

1. See Output Skills K1-K2 on this module's *ID Sheet*. One or both of these skills, or none, may be on the actual exam.
2. See this module's *Problem Supplement*, Problem 1.
3. See this module's *Problem Supplement*, Problem 3.
4. See this module's *Problem Supplement*, Problem 4.

Brief Answers:

1-4. See this module's *text* and *Problem Supplement*.