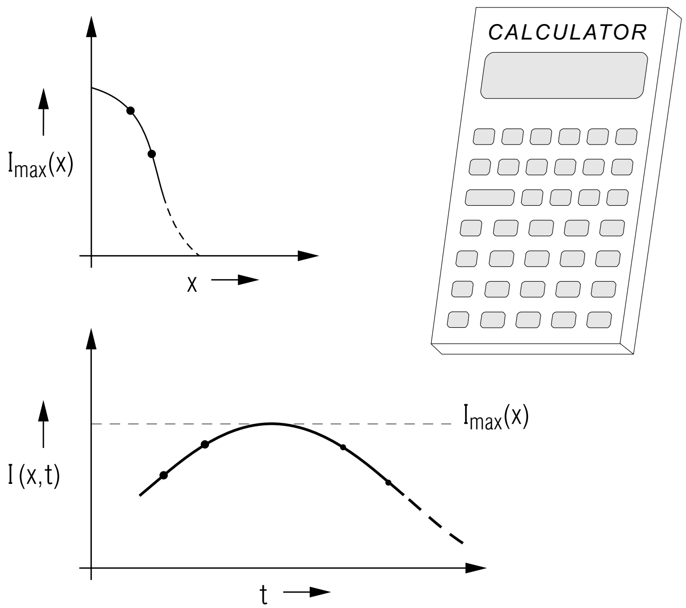
MISN-0-238

CONSTRUCTING AN INTERFERENCE PATTERN - A PROJECT -



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

CONSTRUCTING AN INTERFERENCE PATTERN - A PROJECT -

by

J. S. Kovacs and Peter Signell

Title: **Constructing an Interference Pattern - A Project -**

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Length: 1 hr; 20 pages

Input Skills:

1. Determine the wave disturbance at a given point resulting from two equal coherent wave sources (MISN-0-230).
2. Draw and annotate graphs for good scientific communication (MISN-0-401).

Output Skills (Project):

P1. Use numerical amplitude addition to construct a significant portion of a wave interference pattern (see this module's *Local Guide*).

External Resources (Required):

1. A calculator with square roots and trigonometric functions, and graph paper.

Post-Options:

1. "Interference, Many Sources; Radio Interferometry" (MISN-0-231).

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OF PROJECT PHYSNET

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Our publications are designed: (i) to be updated quickly in response to field tests and new scientific developments; (ii) to be used in both classroom and professional settings; (iii) to show the prerequisite dependencies existing among the various chunks of physics knowledge and skill, as a guide both to mental organization and to use of the materials; and (iv) to be adapted quickly to specific user needs ranging from single-skill instruction to complete custom textbooks.

New authors, reviewers and field testers are welcome.

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1. Interference Patterns Betray Waves

Light's ability to produce striking interference patterns is a prime indicator that it has some sort of wave nature. However, the waves are not so easy to examine more directly. For example, the eye could only see the individual wave arrivals if they came a millionth of a millionth as fast! Coupled with the waves' high speed (over a billion km/hr) and short wavelength (less than a millionth of a meter), we generally find it easier to examine light's wave properties through its interference patterns.

2. Outline of the Project

The idea here is to take a wave incident normally upon two parallel slits cut out of an opaque light barrier, then propagate the light passing through the slits onto a screen. Calculation of the resulting interference pattern on the screen illustrates how interference patterns arise. It also tests one's understanding of wave properties: wave motion, phase, intensity, and average intensity all enter into the calculation in crucial ways. There is also calculator practice in locating a zero of a non-linear function. In fact, if your calculator is programmable you can plot much more of the interference pattern than just the central peak required here. This project, the mathematical construction of wave amplitudes and interference, is a good test of your understanding of waves. If you have trouble, it is a signal that your understanding is inadequate.

3. Procedure

(a) Obtain a trig calculator and graph paper.

(b) Decide on numerical values of the apparatus dimensions shown in Fig. 1. The distances (d,L,x) can be specified in units of wavelength of the incident light. We suggest that d, the slit separation, be of

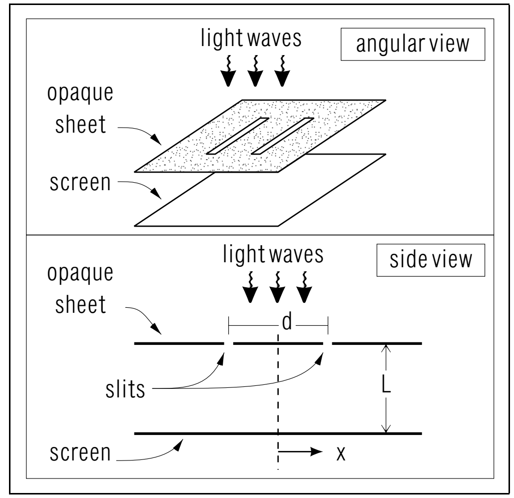


Figure 1. Two views of a 2-slit interference system.

the order of a wavelength. *Help: [S-1]*[1] The slit-to-screen distance L should be at least a few wavelengths; then the first minimum will occur at a modest value of screen position x. Note that the interference pattern will be symmetric about $x = 0$. *Help: [S-2]*

(c) Choose a convenient function to describe the time and position variation of the plane waves that are incident upon the slits. *Help: [S-3]*

(d) Using the function chosen in (c), calculate the peak intensity at $x = 0$ as well as a time at which this peak occurs. We suggest that you measure time in units of the light wave's vibrational period (inverse of frequency). *Help: [S-5]*

(e) Using simple trig, the calculator, and interpolation, determine the first screen position x_0 which has zero intensity at all times. *Help: [S-7]* (Assume that the amplitude of each slit's wave is constant; that it does not fall off with distance from the slit. This simplifying assumption is really only a good approximation when $x \gg L$ but we will use it anyway.)

(f) Choose two or more screen points that nicely divide the interval between $x = 0$ and x_0, the first minimum. Calculate the intensity as a function of time at each of these points, and at $x = 0$ and

[1] [S-1] refers to sequence [S-1] in the attached Special Assistance Supplement. It provides special help, if you need it, for what was just discussed.

x_0, graphing the results in order to find the peak intensity at each point.[2] *Help: [S-6]* State, on the graphs, why these time variations are not seen by the human eye. *Help: [S-4]*

(g) Plot a graph of average intensity (time-average) vs. screen position x, including the values you obtained at $x = 0$, at x_0, and at the two or more points in between. Clearly identify x_0 as well as the other points on your graph. *Help: [S-8]*

(h) For obtaining credit for your project, see this module's attached *Local Guide.*

Acknowledgments

We would like to thank field testers Bill Lane, Steve Smith, and William Francis for their strong feedback on the first version. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

A. Formal Solution

(for those interested)

$$\begin{aligned} A &= A_1 + A_2 \\ &= \cos 2\pi \left(\frac{D}{\lambda} - \frac{t}{T}\right) + \cos 2\pi \left(\frac{D}{\lambda} - \frac{t}{T}\right) \qquad \textit{Help: [S-12]} \\ &= \alpha \cos 2\pi \left(\frac{\overline{D}}{\lambda} - \frac{t}{T}\right) \end{aligned}$$

Where:

$\alpha = 2\ \cos[\pi(D_1 - D_2)/\lambda]$

$D_1 =$ distance from slit #1 to position x on screen. *Help: [S-7]*

$D_2 = \ldots$ slit #2 ...

[2] You can use calculus, if you wish, to determine the times at which the peaks occur.

$\overline{D} = (D_1 + D_2)/2$

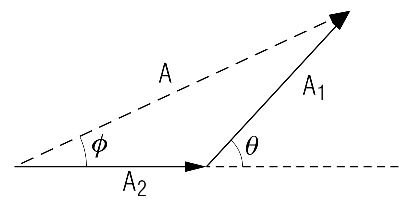
$T =$ light wave period

Then the time-average intensity is:

$$\overline{I} = \alpha^2/2\,.$$

B. Phasor Method

(for those interested)



See side A in the sketch.

See Appendix A for def. of D_1, D_2.

$A = \alpha = (2 + 2\cos\theta)^{1/2} = 2\cos\theta/2$

$\theta = 2\pi \dfrac{D_1 - D_2}{\lambda}$

$\alpha = 2\cos \dfrac{\pi(D_1 - D_2)}{\lambda}$

$\phi = \theta/2$

$$\begin{aligned} \text{(phase of } A) &= \text{(phase of } A_2) + \phi = \text{(phase of } A_1) - \phi \\ &= 2\pi\frac{D_2}{\lambda} + \frac{\theta}{2} = 2\pi\frac{D_2}{\lambda} + \frac{\pi(D_1 - D_2)}{2\lambda} \\ &= \frac{2\pi}{\lambda}\left(\frac{D_1 + D_2}{2}\right) \end{aligned}$$

Hence: $A = A_1 + A_2 = \alpha \cos\left[2\pi\left(\dfrac{D_1 + D_2}{2\lambda} - \dfrac{t}{T}\right)\right]$

Then the time-average intensity is:[3] $\overline{I} = \alpha^2/2$.

[3]For an explanation of this method see "Phasors" (MISN-0-27).

LOCAL GUIDE

Equipment Needed:

You will need a calculator with square roots and trig functions.

Choice of Values:

No credit will be given if your choices for numerical values of d and/or L coincide with those used in the Special Assistance Supplement, sequence [S-1].

Credit for this Module:

Complete all work before coming to our Exam Room.

Bring, to our exam room, the originals of your graphs and your geometry sketch that shows your constants. As usual, fill out your Exam Application sheet and bring it and your project materials to the Exam Manager at the entrance to the Exam Room. The Exam Sheet for Unit 238 will tell you that you must hand in your originals.

SPECIAL ASSISTANCE SUPPLEMENT

S-1 *(from TX-3b)*

For example: $d = 2\lambda$; $L = 3\lambda$

S-2 *(from TX-3b)*

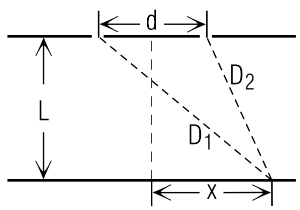
The physical apparatus and the light waves are symmetrical about $x = 0$ so the interference pattern produced by them must have that same symmetry.

S-3 *(from TX-3c)*

We will choose (see the diagram):

$$A_{1,2} = \cos\left[2\pi\left(\frac{D_{1,2}}{\lambda} - \frac{t}{T}\right)\right].$$

We could equally well have chosen a sine and cosine function, or either of them with a phase.



Note: T is wave "period." With our cosine choice, at position x from each of the slits, the amplitudes are:

$$A_1 = \cos\left[2\pi\left(\frac{D_1}{\lambda} - \frac{t}{T}\right)\right]$$

$$A_2 = \cos\left[2\pi\left(\frac{D_2}{\lambda} - \frac{t}{T}\right)\right]$$

where:

$$\frac{D_1}{\lambda} = \left[\left(\frac{L}{\lambda}\right)^2 + \left(\frac{x + d/2}{\lambda}\right)^2\right]^{1/2}$$

$$\frac{D_2}{\lambda} = \left[\left(\frac{L}{\lambda}\right)^2 + \left(\frac{x - d/2}{\lambda}\right)^2\right]^{1/2}$$

S-4 *(from TX-3f)*

At each point on the screen the intensity I is fluctuating at about 10^{15} times per second. The eye cannot respond this fast so it just registers the average intensity, which is exactly one-half the peak intensity. (This is because the total amplitude is a cosine wave so the intensity is a cosine-squared function. The latter's average value is half its peak values).

S-5 *(from TX-3d)*

$x = 0$. Then:

$$\frac{D_1}{\lambda} = \frac{D_2}{\lambda} = \left[\left(\frac{L}{\lambda}\right)^2 + \left(\frac{d/2}{\lambda}\right)^2\right]^{1/2}$$

hence $A_1 = A_2$ and the total amplitude is just twice either one. The peak value of either one is unity so the peak value of the total amplitude is 2. It occurs when each amplitude has a peak; that is, when the cosine arguments vanish. This time is:

$$\frac{t_{peak}}{T} = \frac{D_1}{\lambda} = \left[\left(\frac{L}{\lambda}\right)^2 + \left(\frac{d/2}{\lambda}\right)^2\right]^{1/2}$$

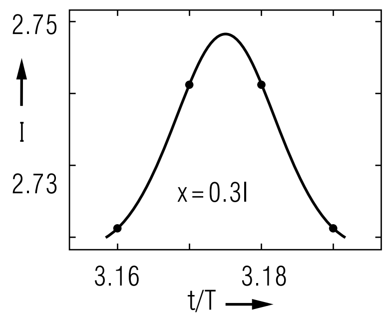
For the S-1 case,

$t_{peak} = \sqrt{10}\,T \approx 3.16\,T$

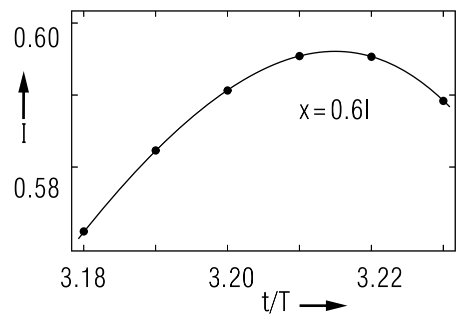
$\boxed{\text{S-6}}$ *(from TX-3f)*

Using $\boxed{\text{S-1}}$ plus the fact that the light intensity I is the square of the total amplitude A, we find at points $x = 0.3\lambda$ and $x = 0.6\lambda$:
$I = |A|^2 = |A_1 + A_2|^2$

x/λ	t/T	I
0.3	3.16	2.724
	3.17	2.745
	3.18	2.746
	3.19	2.724
	[S-9]	



x/λ	t/T	I
0.6	3.18	.574
	3.19	.587
	3.20	.595
	3.21	.599
	3.22	.599
	3.23	.593



Thus:
$I_{peak}(x = 0.3\lambda) = 2.750$
$I_{peak}(x = 0.6\lambda) = 0.600$

$\boxed{\text{S-7}}$ *(from TX-3e and Appendix A)*

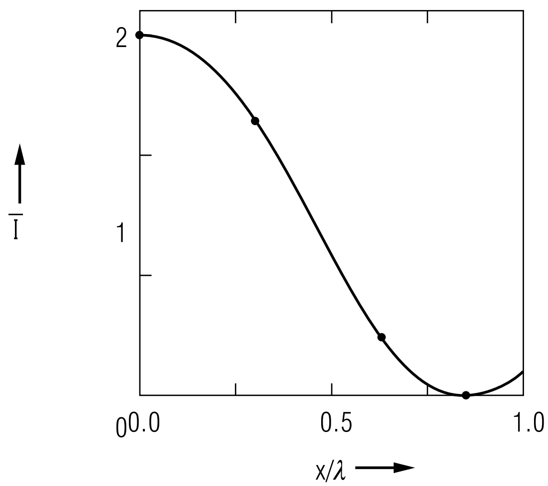
The null will occur when:
$D_1 - D_2 = \lambda/2$
so the two slit amplitudes are then:
$\frac{D_1}{\lambda} - \frac{D_2}{\lambda} = 0.5$
Assuming the same constants as in $\boxed{\text{S-1}}$,
$$0.5 = \left[9 + \left(\frac{x_0}{\lambda} + 1\right)^2\right]^{1/2} - \left[9 + \left(\frac{x_0}{\lambda} - 1\right)^2\right]^{1/2}$$
Solve by interpolation (faster than algebra, but use algebra if you wish! [S-13])
Note: RHS $\equiv$ "Right Hand Side of the equation."

x/λ	RHS	Differences
.650	.403	
		> .030
.700	.433	
		> .029
.750	.463	
		> .029
.800	.492	
		> .029
.850	.521	

Constant differences mean that the RHS is linear in (x/λ). We can interpolate graphically or algebraically. Algebraically:
$\frac{x_0}{\lambda} = 0.800 + (0.008 \times \frac{0.050}{0.029}) = 0.814$ [S-10]
Thus: $A(x = 0.814\lambda, \text{any time}) = 0$. Graphical interpolation gives the same answer, of course. *Help: [S-14]*

$\boxed{\text{S-8}}$ *(from TX-3g)*

See *Help: [S-4]*, *Help: [S-7]*, *Help: [S-6]* for these four points.
See *Help: [S-2]* for a discussion of why the slope is sketched as zero at the origin.

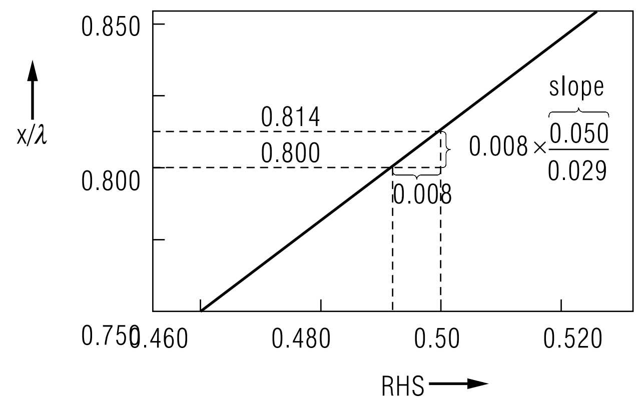


See *Help: [S-4]* for why $\overline{I}_{max} = 2$, not 4.

$\boxed{\text{S-9}}$ *(from [S-6])*

The values of x/λ, d/λ, and L/λ were used to get D_1/λ and D_2/λ. These in turn were used to get A_1 and A_2, hence I, for various times t. Should you use radians or degrees? Does it matter? *Help: [S-11]*

$\boxed{\text{S-10}}$ *(from [S-7])*



This can also be found using the straight line equation. *Help: [S-13]*

$\boxed{\text{S-11}}$ *(from [S-9])*

It sure does matter! The "2π" in the equation denominators are really "2π radians." Degrees would be incorrect.

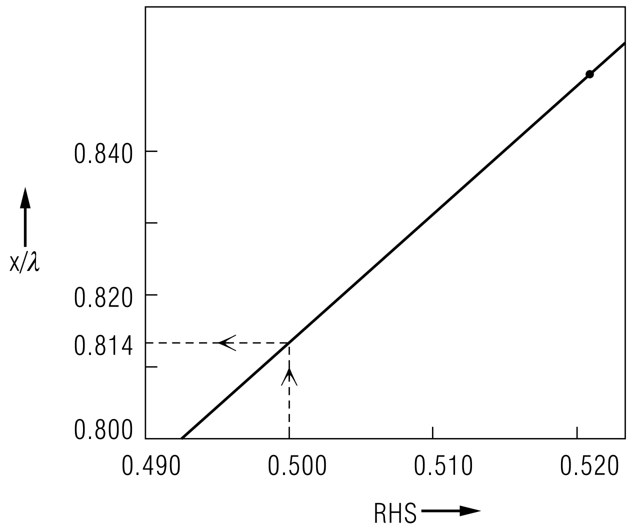
$\boxed{\text{S-12}}$ *(from Appendix A)*

See Input Skill #1 for the reference.

$\boxed{\text{S-13}}$ *(from [S-10])*

To find x/λ brainlessly, find the equation of the straight line:
$\left(\frac{x}{\lambda}\right) = (\text{slope}) \cdot (\text{RHS}) + (\text{intercept})$.
Do it by substituting two known points:
$0.800 = (\text{slope})\,(0.492) + (\text{intercept})$;
$0.850 = (\text{slope})\,(0.521) + (\text{intercept})$.
Solve those two equations in two unknowns, simultaneously, to get:
$\text{slope} = \frac{.850 - .800}{.521 - .492} = \frac{.050}{.029}$;
$\text{intercept} = (.800) - (.492)(\frac{.050}{.029})$.
Then put those into the desired RHS:
$$\left(\frac{x}{\lambda}\right) = \left(\frac{.050}{.029}\right) \cdot (.500) + (.800) - (.492) \cdot \left(\frac{.050}{.029}\right)$$

S-14 *(from [S-7])*



MODEL EXAM

INSTRUCTIONS TO THE GRADER

Grader! The student must have attached:

- the ORIGINAL of his/her graphs
- the ORIGINAL of his/her geometry sketch and constants

If the student handed in a copy, not an original, of either one, then

immediately give the student a grade of zero

on this exam. Write the reason on the Exam Answer Sheet and grade no further.