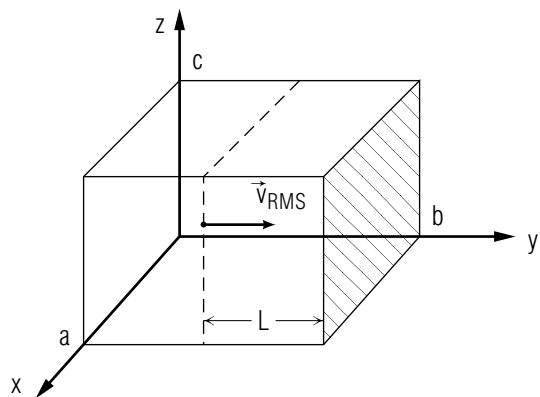# PHYSNET

MISN-0-157

TEMPERATURE AND PRESSURE OF AN IDEAL GAS: THE EQUATION OF STATE

TEMPERATURE AND PRESSURE OF AN IDEAL GAS: THE EQUATION OF STATE

by

William C. Lane

Michigan State University

1. Introduction

2. Temperature of an Ideal Gas

3. Average Pressure of an Ideal Gas

4. Equation of State of an Ideal Gas

Title: **Temperature and Pressure of an Ideal Gas: The Equation of State**

Author: William C. Lane, Michigan State University

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Length: 1 hr; 24 pages

Input Skills:

1. Vocabulary: center of mass (MISN-0-16) or (MISN-0-413), frame of reference (MISN-0-11) or (MISN-0-412), kinetic energy, joule (MISN-0-20) or (MISN-0-414).
2. Calculate a particle's change in momentum when it undergoes an elastic collision (MISN-0-15) or (MISN-0-413).

Output Skills (Knowledge):

K1. Vocabulary: absolute temperature scale, Boltzmann's constant, Celsius, equation of state, Fahrenheit, ideal gas, Kelvin, mole, root mean square (RMS) speed, pressure, average pressure, temperature, thermal equilibrium, thermometer.

K2. Starting from Newton's second law, derive the expression for the pressure the molecules of an ideal gas exert on the walls of a container in terms of their root mean square (RMS) speed.

K3. State the relation between temperature and ideal-gas average kinetic energy expressed in terms of the RMS speed of the gas molecules.

K4. Derive the ideal-gas equation of state, starting from microscopic definitions of temperature and pressure.

Output Skills (Problem Solving):

S1. Given a temperature from one temperature scale (Celsius, Fahrenheit, Kelvin) compute the corresponding temperature for the other scales.

S2. Given the known macroscopic properties of an ideal gas at one or more different states, use the equation of state to calculate the unknown properties at another state.

S3. Given a distribution of particles with known masses and velocities, calculate the RMS speed, average kinetic energy, and temperature.

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1. Introduction

1a. Statistical Methods are Necessary for Crowded Systems.
When dealing with the mechanics of systems of very large numbers of particles it is necessary to use statistical methods. It's obviously impossible to treat individually each of the molecules of a gas consisting of 10^{20} particles, even if you think you know the laws of physics which determine the motion of each of these particles. Even the most sophisticated computers currently conceived would be hopelessly swamped trying to keep track of the forces acting on each of the 10^{20} particles moving about in the gas, as well as the position, velocity, and acceleration of each. Actually, even if you could obtain all of this detailed information, only summaries or averages of the various quantities would have useful meaning. For this reason statistical mechanical methods are not only the sole practical way, they are the most useful way of dealing with the dynamics of such systems.

1b. Microscopic Descriptions Yield Macroscopic Properties.
The averages of the kinematical and dynamical quantities obtained by statistically describing a microscopic system must be related to measurable quantities. For gases these usually are the volume[1] occupied by the gas, the pressure,[2] temperature, and the number of molecules involved, which together determine the state of the system. For an ideal gas, the simplest model of intermolecular forces is assumed: there are no interactions between the molecules unless their centers happen to coincide, in which case they bounce off one another like hard spheres, and the molecules are assumed to be point masses. For such a model, the equation of state, that is, the relation between the measurable quantities (pressure, volume, temperature), can be derived in a straightforward way once temperature and average pressure are defined.

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[1]For a definition of this term see this unit's *Glossary*.
[2]For a definition of this term see this unit's *Glossary*.

2. Temperature of an Ideal Gas

2a. Macroscopic Viewpoint. Most of us use the macroscopic viewpoint of temperature in our daily lives. To us, "temperature" has always been simply a measuring scale to indicate how "hot" or how "cold" something is. We know that temperature is measured with a device known as a "thermometer." We know that "heat" travels from points of high temperature to points of low temperature. However, such a viewpoint is of limited usefulness in describing the temperature of a system of many particles. Before we proceed we must develop a microscopic picture of the system.

2b. Microscopic Viewpoint: An Ideal Gas. In order to illustrate the microscopic view of a large system of particles, we will choose the simplest system we can imagine, an ideal gas. An ideal gas is one in which the intermolecular forces are negligible, and the molecules can be treated as point masses which obey Newton's three laws of motion. The energy of the gas molecules is entirely translational kinetic energy. Temperature is defined as a quantity directly proportional to the average kinetic energy of a molecule, in the center of mass frame of reference.

For a system of N particles whose masses are m_1, m_2, m_3, ... m_N and whose velocities are $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$, ... $\vec{v}_N$, the average kinetic energy of the particles is:

$$E_{k,av} = \frac{1}{N} \left[\sum_{i=1}^{N} \frac{1}{2} m_i v_i^2 \right]. \tag{1}$$

If all the particles have the same mass m, then Eq. (1) becomes:

$$E_{k,av} = \frac{1}{2} m \left[\frac{1}{N} \sum_{i=1}^{N} v_i^2 \right].$$

A quantity known as the "root mean square" speed of the particles, v_{RMS}, is defined as:

$$v_{\text{RMS}} = \sqrt{v_{\text{RMS}}^2} = \sqrt{\frac{1}{N}(v_1^2 + v_2^2 + \ldots + v_N^2)} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} v_i^2}. \tag{2}$$

In the "RMS" notation, the average kinetic energy of a particle in this system of N identical particles, each with mass m, is:

$$E_{k,av} = \frac{1}{2} m v_{\text{RMS}}^2. \tag{3}$$

2c. Thermal Equilibrium. For Eq. (3) to be of any use in defining temperature, we must require that the gas molecules have a completely random motion, a condition known as "thermal equilibrium." At thermal equilibrium, the system has no "memory" of how it was prepared and the only forces acting on the particles are the contact forces felt during collisions with the walls of the gas container and with other molecules. If the system is at thermal equilibrium, the temperature may be defined as a quantity directly proportional to the average kinetic energy of the gas molecules:

$$E_{k,av} = \frac{1}{2}mv_{\mathrm{RMS}}^2 = \frac{3}{2}kT.$$

so:

$$T = \frac{mv_{\mathrm{RMS}}^2}{3k}. \tag{4}$$

The constant k is called "Boltzmann's constant." The numerical value of k is 1.3805×10^{-23} J/K, where K is a unit of temperature on the Kelvin absolute temperature scale. On this scale, ice at normal atmospheric pressure melts at 273.15 K, and water at normal atmospheric pressure boils at 373.15 K. If the system is not in thermal equilibrium then temperature is not a meaningful property of the system.

2d. Other Temperature Scales. Several other scales are commonly used for measuring temperatures and will be defined. The Celsius (formerly centigrade) temperature scale is defined by:

$$T_C = (T_K - 273.15) \,^{\circ}\mathrm{C}, \tag{5}$$

where T_C is the Celsius temperature, T_K is the Kelvin temperature, and the Celsius unit of temperature is a "degrees Celsius." A change in temperature of 1 K is exactly equal to a change in temperature of $1\,^{\circ}$C. On this scale, water boils at $100\,^{\circ}$C and freezes at $0\,^{\circ}$C. Also used, especially in day-to-day life, is the Fahrenheit temperature scale. The conversion of temperature from a Celsius scale to a Fahrenheit scale is quite straightforward and can be easily derived without memorizing formulas. Realizing that the two temperature scales are linearly related, we may write

$$T_F = a(T_C) + b, \tag{6}$$

and merely solve for a and b. This can be done by evaluating Eq. (6) at two temperatures where both the Fahrenheit and Celsius values are known: (1) at the freezing point of water: $32\,^{\circ}\mathrm{F} = a(0\,^{\circ}\mathrm{C}) + b$; and (2) at the boiling point of water: $212\,^{\circ}\mathrm{F} = a(100\,^{\circ}\mathrm{C}) + b$. The first relation determines b as $32\,^{\circ}$F, and by using this value in the second relation, a is

seen to be $1.8\,^{\circ}\mathrm{F}/\,^{\circ}\mathrm{C}$. Converting 1.8 to its fractional equivalent of 9/5, we obtain:

$$T_F = (9/5)T_C + 32\,^{\circ}\mathrm{F}, \tag{7}$$

equivalently:

$$T_C = (5/9)(T_F - 32\,^{\circ}\mathrm{F}). \tag{8}$$

3. Average Pressure of an Ideal Gas

3a. Macroscopic Definition of Pressure and its Units. For macroscopic systems such as a fluid, pressure is defined as the force per unit area exerted by the fluid on some surface:

$$P = \frac{F}{A} \tag{9}$$

where F and thus P may vary continuously over A, the surface area.[3] units of pressure in the MKS system are newtons per square meter ($\mathrm{N/m^2}$) or joules per cubic meter ($\mathrm{J/m^3}$). Other frequently used units of pressure include the atmosphere (atm), pounds per square inch ($\mathrm{lb/in^2}$ or psi), pounds per square foot ($\mathrm{lb/ft^2}$) and millimeters of mercury ($\mathrm{mm\,Hg}$ or torr). The last unit of pressure is based on a method of specifying air pressure by means of measuring the height of a column of mercury in a barometer. The conversion factors between the various systems of units listed above are:

$$1\,\mathrm{atm} = 1.013 \times 10^5\,\mathrm{N/m^2} = 760\,\mathrm{torr}$$
$$1\,\mathrm{lb/in^2} = 144\,\mathrm{lb/ft^2} = 6891\,\mathrm{N/m^2}$$

3b. Microscopic Definition of Average Pressure. For a system consisting of discrete microscopic gas molecules, we must think in terms of the average pressure exerted by the molecules as they collide elastically with the walls of their container. By Newton's second law, the average force exerted by one object on another during a collision is

$$\vec{F}_{\mathrm{ave}} = \frac{\Delta\vec{p}}{\Delta t}, \tag{10}$$

where $\Delta\vec{p}$ is the momentum exchanged during a collision time interval of Δt. We will use this average force to calculate the average pressure of an ideal gas in some container of volume V.

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[3]For a further discussion see "Archimede's Principle; Bernoulli's Theorem" (MISN-0-48).
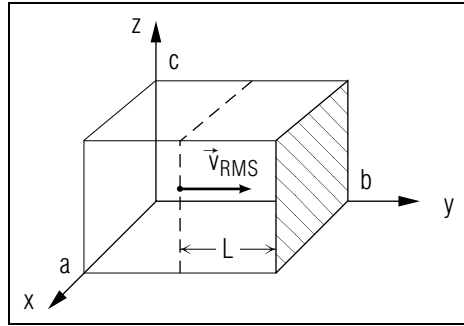
Figure 1. An idealized system used to derive the pressure of an ideal gas.

3c. The Statistical Model of the System. We are interested in systems that have reached thermal equilibrium, and at thermal equilibrium the gas molecules are in completely random motion. Also, in order for statistical mechanics to be valid, our system must contain a very large number of molecules. Random motion implies that the molecules are moving in all directions, and the large number of molecules implies both that all directions and allowable speeds[4] are represented and also that the randomness is maintained. There is no net movement in any direction.

3d. Calculating Statistical Averages. When we calculate macroscopic quantities like pressure and temperature, we are dealing with averages over the many molecules in the system. For ease in computation we can assume from the beginning that all of the molecules move with the same average speed. In order for the net velocity to be zero, then, the number of molecules moving in any one direction must equal the number of molecules moving in the opposite direction. In particular, we could resolve the velocity of each molecule into its three components along the x, y and z axes and use the fact that the sum of the components along any one of the six positive and negative axes must equal the sum along each of the other five. Equivalently, however, we could consider the molecules to be traveling only in the direction of one or another of the axes, with the condition that the number of molecules moving in any one direction must equal the number moving in any other direction. Thus for the sake of calculating macroscopic quantities, we may make the assumption that all of the molecules move with the same average speed and $1/6$ of them move along each of the six positive and negative axes. Temperature is defined directly in terms of the molecules' RMS speed, so it is this "average speed" that we use.

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[4]Allowable speeds are those consistent with the temperature of the system. See "Energy Distribution Functions: Boltzmann Distribution" (MISN-0-159).

3e. Derivation of Average Pressure. The average pressure of an ideal gas may be derived by considering an imaginary system consisting of a rectangular parallelopiped with side lengths a, b, and c, oriented as shown in Fig. 1. N hard spheres (atoms) of negligible size are contained inside this box. The atoms are moving randomly, and after thermal equilibrium is established we can consider that the RMS speed of these atoms remains fixed. A particular atom moving in the positive y-direction with speed v_{RMS} is instantaneously at a distance L from the shaded face of the volume. It will strike the face in time Δt given by:

$$\Delta t = L/v_{\text{RMS}} \tag{11}$$

In that time, all of the other atoms moving in the positive y-direction that were within the sub-volume defined by L and the shaded face (of area A) will also strike this face. By the law of conservation of momentum, each atom will impart a momentum $2mv_{\text{RMS}}$ to the wall. The number of these collisions in time Δt is proportional to the subvolume:

$$N_{\text{collisions}} = \frac{LA}{V}\,\frac{N}{6} \tag{12}$$

where $N/6$ is the total number of atoms moving in the $+y$-direction, and (LA/V) is the fraction of the total volume V in which these atoms are contained. The total momentum exchanged with the wall is therefore:

$$(\Delta p)_{\text{Total}} = N_{\text{collisions}}\,(2mv_{\text{RMS}}) = \frac{1}{3}\,\frac{N\,L\,A\,m\,v_{\text{RMS}}}{V}. \tag{13}$$

Substituting this expression for the momentum transferred to the wall of the container into Eq. (10), we obtain:

$$F_{\text{ave}} = \frac{\Delta p}{\Delta t} = \frac{\frac{1}{3}\,\dfrac{N\,L\,A\,m\,v_{\text{RMS}}}{V}}{L/v_{\text{RMS}}}$$

or

$$F_{\text{ave}} = \frac{N\,A\,m\,v_{\text{RMS}}^2}{3V}. \tag{14}$$

Thus the average pressure of the gas is:

$$P = \frac{1}{3}\,\frac{N}{V}\,mv_{\text{RMS}}^2. \tag{15}$$

4. Equation of State of an Ideal Gas

4a. Definition of State. In general, the state[5] of a system is defined by specifying a limited number of properties of the system and leaving other properties unspecified. The state of a gas is completely specified by a knowledge of three of its macroscopic properties: pressure, temperature and volume.[6] In formal mathematical language, the equation can be written:

$$F(P, T, V, m) = 0. \tag{16}$$

By combining the expression for the pressure and temperature of an ideal gas and eliminating the term $mv_{\rm RMS}^2$, we may express the equation of state of an ideal gas as:

$$PV = NkT \tag{17}$$

where P is the average pressure of the gas, T is the temperature of the gas, V is the volume of the gas container, N is the number of molecules (or atoms) of the gas, and k is Boltzmann's constant.

4b. Alternate Forms of the Equation of State. The equation of state of an ideal gas can be expressed in several other forms, each of which involve pressure and temperature. We can express the equation of state in terms of the number of moles of gas by using the relation

$$n = \frac{N}{N_A}, \tag{18}$$

where n is the number of moles of gas, N is the number of molecules of gas, and N_A is Avogadro's number, 6.023×10^{23}. Thus $N = nN_A$, so

$$PV = n(N_A k)T = nRT, \tag{19}$$

where we have introduced a new symbol R, called the "ideal gas constant":

$$R = \left(6.023 \times 10^{23}\,{\rm mole}^{-1}\right) \times \left(1.3805 \times 10^{-23}\,{\rm J/K}\right)$$

so

$$R = 8.3143\,{\rm J/(K\,mole)}. \tag{20}$$

Another form of the equation of state can be obtained by converting the number of moles of gas to the equivalent mass using the expression:

$$n = \frac{m}{M}, \tag{21}$$

_____

[5]The term "state" as used here implies an equilibrium state: temperature and pressure are the same at all points.

[6]An equation of state refers to a particular gas; the mass of the gas or the number of gas molecules present is implicitly specified in such an equation.

where m is the mass of the ideal gas in its container and M is the molecular weight of the gas. Substituting Eq. (21) into Eq. (19) and dividing each side by V we obtain:

$$P = \frac{mRT}{MV} = \rho RT/M, \tag{22}$$

where ρ is the density of the gas.

4c. Using the Equation of State. The equation of state is a powerful tool we can use to calculate the unknown macroscopic properties of an ideal gas as it changes its state. For any particular system, we identify which quantities are constant in time and which are variables. We then rearrange the equation of state to get all variables on one side of the equality and all constants on the other side. Since the "constants" side of the equation is certainly constant in time, the particular combination of variables on the other side of the equality must also be constant in time.

4d. Example: A Closed System. We illustrate use of the equation of state by treating an ideal gas system that is "closed," i.e. for which no molecules enter or leave the container. Then N is the constant in Eq. (17) so we rewrite that equation this way:

$$\frac{PV}{T} = Nk = nR = \text{ constant.} \tag{23}$$

Thus the pressure, temperature and volume of the gas at one time, called state 1, are related to the corresponding properties at another time, called state 2, by the relation:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} = \text{ constant.} \tag{24}$$

Note that we do not need to know the constant N (or n) if only one property is unknown in one of the states. For example, given the pressure and volume in both states and the temperature in state 1, the temperature in state 2 is:

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right) \left(\frac{V_2}{V_1}\right).$$

Note that most of the units will cancel so a conversion of units may not be needed. Also note that absolute (e.g. the Kelvin scale) temperatures must be used with the equation of state.

Acknowledgments

Glossary

- **absolute temperature scale**: a temperature scale that assigns a temperature of zero to the lowest possible state of molecular energy. The Kelvin scale is an example of an absolute temperature scale.

- **Boltzmann's constant**: a constant, denoted by k, used to relate temperature to molecular energy. For an ideal gas, $E_{k,ave} = (3/2) \, k \, T$.

- **Celsius**: a temperature scale with increments the same size as the Kelvin scale but which assigns zero temperature to the freezing point of water. The boiling point of water is assigned to 100°C. This range of temperatures defines the size of a degree Celsius. It is not an absolute scale of temperature.

- **density**: the mass per unit volume at any space point. It is often written $\rho(x, y, z)$ or, equivalently, $\rho(\vec{r})$. The density of pure water at 4 °C is $\rho = 1 \, \mathrm{gram/cm^3}$ down to depths where pressure causes compression and a rise in density.

- **equation of state**: a mathematical expression which includes a limited number of macroscopic properties of a system and completely specifies the system.

- **Fahrenheit**: a temperature scale used commonly in English-speaking countries. This scale assigns the temperature 32 °F to the freezing point of water, and 212 °F to water's boiling point. It is not an absolute scale of temperature.

- **ideal gas**: a system composed of particles which have negligible volume and interact with each other only by collisions. The particles have no structure and function as simple point masses. Real gases behave like ideal gases at sufficiently low pressures and densities.

- Kelvin: an absolute scale of temperature with increments the same size as those of the Celsius scale. On this scale, water freezes at 273 K and boils at 373 K.

- **mole**: an "Avogadro's number" of anything, i.e. 6.023×10^{23} of anything; like a dozen eggs or a gross of oranges, we say "a mole of gas molecules."

- **root mean square (RMS) speed**: the speed of a particle in a statistical system which has a kinetic energy equal to the average kinetic energy of the system. The RMS speed is found by averaging the square of all particle speeds in the system and taking the square root of this mean.

- **temperature**: a macroscopic property of a system which may be statistically related to the average kinetic energy of the system (or the root mean square speed).

- **thermal equilibrium**: the state of a statistically described system where the motion of the particles is completely random and temperature is a well defined quantity.

- **thermometer**: a device used to measure temperatures. Thermometers operate via a variety of mechanisms whereby some property of a material which depends on temperature is in thermal contact with the system. Examples include the thermal expansion of liquids and gases and the electrical resistivity of a wire.

- **volume**: the amount of space inside a closed surface, measured in cubic length units. For example, the volume inside a cube with dimensions $2 \, \mathrm{m} \times 2 \, \mathrm{m} \times 2 \, \mathrm{m}$ is $8 \, \mathrm{m^3}$ ("eight cubic meters"). The volume enclosed by a sphere of radius 3 m is $36\pi \, \mathrm{m^3}$.

PROBLEM SUPPLEMENT

Note: Problems 12 and 13 also occur in this module's *Model Exam.*

1.
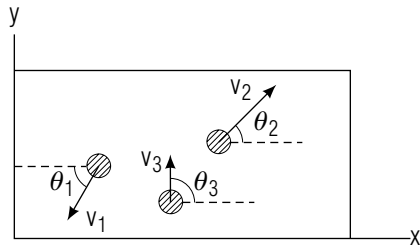
$v_1 = 40\,\text{ft/sec}, \theta_1 = 60°$
$v_2 = 20\sqrt{2}\,\text{ft/sec}, \theta_2 = 45°$
$v_3 = 20(\sqrt{3} - 1)\,\text{ft/sec}, \theta_3 = 90°$

An x-y coordinate system is set up at one of the corners of a billiard table in the plane of the table. At a given instant, three billiard balls are observed to be moving with the speeds and directions indicated in the sketch above. The dotted lines are parallel to the x-axis.

a. What is the average value of v_y, the y-component of the velocity?

b. What is the average value of v_x?

c. Calculate the RMS value of v_x for these billiard balls. *Help: [S-4]*

d. Calculate the RMS value of v_y.

e. Calculate the RMS value of v.

f. If the weight of each billiard ball is one pound (lb), what is the average kinetic energy of these billiard balls? *Help: [S-1]*

2. Consider a simple system of two particles, each of mass M. First we have them in a situation labeled case A:
$\vec{v}_1 = +(3\,\text{m/s})\,\hat{x}$
$\vec{v}_2 = -(3\,\text{m/s})\,\hat{x}$.
Then we have them in a situation labeled case B:
$\vec{v}_1 = -(53\,\text{m/s})\,\hat{x}$
$\vec{v}_2 = -(47\,\text{m/s})\,\hat{x}$.
In which case does the system have a higher temperature?

3. If an ideal gas with molecules the same mass as molecular hydrogen have an RMS speed of $6.08\,\text{m/s}$, what is the temperature of the gas? What should be the RMS speed of this ideal gas for the temperature to be nearer to room temperature, say $300\,\text{K}$? *Help: [S-2]*

4. Suppose we have two gases, both consisting of the same total number of particles, both ideal, both with the same average kinetic energy. One of the gases consists of N particles with the same mass as diatomic hydrogen; the other consists of N particles with the mass of the electron. Which gas would be at the higher temperature?

5. Suppose that, in Problem 4, one gas (H_2) consisted of N particles, the other gas (electrons) consisted of $100\,N$ particles, while all the other properties mentioned in Problem 4 were the same. Which gas would be at the higher temperature?

6. If the RMS speed of the molecules of a certain gas is kept constant while the number of molecules in the gas is doubled, what happens to each of the following quantities? (Assume the gas is ideal and its is volume constant.)

a. the average kinetic energy;

b. the temperature of the gas;

c. the pressure of the gas.

7.
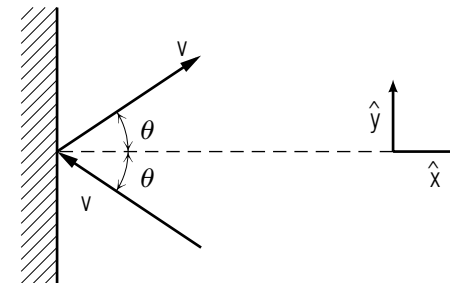
A hard sphere of mass M makes an elastic collision with a rigid wall. Before the collision the velocity of the sphere is $\vec{v}$, making an angle θ with the x-direction. After the collision its speed is still v, but the velocity $\vec{v}$ makes an angle θ reflected about the x-axis from its incident direction, as shown in the sketch.

a. What is the x-component of the incident velocity in terms of v and θ?

b. What is the x-component of the velocity of the rebounding sphere?

c. What is the y-component of the incident velocity?

d. What is the y-component of the velocity of the rebounding sphere?

e. What is the change in the x-component of the velocity as a result of the collision?

f. What is the change in the y-component of the velocity as a result of the collision?

g. What is the change in momentum of the sphere?

h. During an interval of time, Δt, which includes the interval during which the sphere rebounded from the wall, what is the average force that the sphere exerts on the wall?

i. Suppose there are many such spheres in this region making collisions with the wall, all with incident x-component of velocity $-v_x$. If these spheres are distributed so that there are N_x of them per unit length of distance in the x-direction, all headed left, how many of these strike the wall per second? How many strike the wall in time interval Δt? What is the average force exerted on the wall?

8. Room temperature is usually taken to be $20\,°C$. Calculate the corresponding Fahrenheit value of this temperature.

9. A normal human body temperature is usually assumed to be $98.6\,°F$. What is this temperature in degrees Celsius?

10. Scuba divers use tanks of compressed air strapped to their backs to breathe underwater. A regulator attached to the tank feeds air to the diver at the same pressure as the ambient water so that pressures will be balanced throughout the diver's body. The pressure of the water is given by: $P = P_0 + \rho g h$, where P_0 is atmospheric pressure at sea level $(P_0 = 1\,\text{atm} = 1.013 \times 10^5\,\text{N/m}^2)$, ρ is the density of water, g is the acceleration of gravity, and h is the depth of immersion. As a shorthand method, divers use the fact that each $33\,\text{ft}$ of depth increases the pressure by 1 atmosphere of pressure above atmospheric pressure. If a scuba diver were to ascend from a 33 foot depth to the surface without exhaling (i.e. holding his breath) calculate the change in volume of the air in his body, assuming air to be an ideal gas. Assume that the temperature of the air remains the same.

11. When you fill up your car's tires with air at the service station, you usually set the pressure at some fixed value established by the manufacturer. However during subsequent driving the tires heat up due

to friction with the road surface. Assuming the tire has no leaks and is not changing shape appreciably, calculate the new tire pressure when the temperature of the air in the tires increases by $10\,°C$. Assume that the tires initially were inflated to $25\,\text{psi}$ at a temperature of $68\,°F$. *Help: [S-3]*

12. Four particles, each of mass $0.25\,\text{kg}$, have these velocities:
$\vec{v}_1 = (3\,\text{m/s})\,\hat{x} - (7\,\text{m/s})\,\hat{z}$
$\vec{v}_2 = (4\,\text{m/s})\,\hat{y}$
$\vec{v}_3 = (7\,\text{m/s})\,\hat{z}$
$\vec{v}_4 = -(3\,\text{m/s})\,\hat{x} - (4\,\text{m/s})\,\hat{y}$

a. Find the RMS speed of the particles in this system.

b. Calculate the average kinetic energy and "temperature" of the system, assuming it is in thermal equilibrium.

13. An air cylinder initially at room temperature $(68\,°F)$ contains $5.0\,\text{kg}$ of gas at a pressure of $2000\,\text{psi}$. The cylinder then is attached to a compressor and filled to its maximum capacity with a final pressure and temperature of $3000\,\text{psi}$ and $77\,°F$. Calculate the additional mass of air added to the cylinder, assuming air is an ideal gas. *Help: [S-5]*

Brief Answers:

1. a. Zero

   b. Zero

   c. 16.33 ft/s

   d. 24.59 ft/s

   e. 29.52 ft/s

   f. 13.61 ft lb

2. Both have the same temperature. In case B, the center of mass of the particles has a velocity of $-50\,\text{m/s}\,\hat{x}$, so that in the center of mass frame of reference both particles have speed $3\,\text{m/s}$.

3. $3.0 \times 10^{-3}\,\text{K}$; $v_{\text{RMS}} = 1.93 \times 10^3\,\text{m/s}$ at $T = 300\,\text{K}$.

4. T is the same because $E_{\text{k,ave}}$ is the same.

5. T is the same because $E_{\text{k,ave}}$ is the same.

6. a. It remains the same.

   b. It remains the same.

   c. It is doubled.

7. a. $-v\cos\theta$

   b. $+v\cos\theta$

   c. $+v\sin\theta$

   d. $+v\sin\theta$

   e. $+2v\cos\theta$

   f. Zero

   g. $(2Mv\cos\theta)\,\hat{x}$

   h. $(-2Mv\cos\theta/\Delta t)\,\hat{x}$

   i. $N_x v_x$; $N_x v_x \Delta t$; $(-2MN_x v_x^2)\,\hat{x}$

8. $68\,°\text{F}$

9. $37.0\,°\text{C}$

10. The gas doubles in volume.

11. $26\,\text{psi}$ (a change of 1 psi).

12. a. $6.08\,\text{m/s}$

    b. $E_{\text{k,ave}} = 4.62\,\text{J}$, $T = 2.23 \times 10^{23}\,\text{K}$

13. $2.4\,\text{kg}$

SPECIAL ASSISTANCE SUPPLEMENT

S-1 *(from PS, problem 1f)*

The mass of the billiard balls may be related to the weight by recalling the relation $W = mg$. If British units such as "lbs" are used in a calculation, use $g = 32\,\text{ft/s}^2$. Thus if an object has weight $W = 1\,\text{lb}$, then it has mass $m = (1/32)\,\text{lb}\,\text{s}^2/\,\text{ft}$.

S-2 *(from PS, problem 3)*

Hydrogen is a diatomic gas, so its molecular mass is very nearly the mass of 2 protons, i.e. $m = 2.0159\,\text{u}$, and $1\,\text{u}$ (atomic mass unit)$= 1.660 \times 10^{-27}\,\text{kg}$.

S-3 *(from PS, problem 11)*

Remember that to use the equation of state you must specify temperature on an absolute scale, e.g. Kelvin.

S-4 *(from PS, problem 1c)*

If you really can't figure out what to do, follow a professional's way of dealing with such situations:

- Review just what RMS means.

- Reread the description of RMS in the text and in the Glossary.

- Think about the words the abbreviation RMS stands for.

- Notice that the words describe exactly what you do mathematically!

- Do what you think might work in a case with a known answer.

S-5 *(from PS, problem 13)*

The initial and final volumes cancel. The temperatures must be changed to Kelvin. The pressure units cancel.

MODEL EXAM

$$
\begin{aligned}
k &= 1.3805 \times 10^{-23}\,\text{J/K} \\
R &= 8.3143\,\text{J/(K mole)} \\
1\,\text{atm} &= 1.013 \times 10^5\,\text{N/m}^{-2} = 14.70\,\text{lb/in}^2
\end{aligned}
$$

1. See Output Skills K1-K4 in this module's *ID Sheet*. One or more of these skills, or none, may be on the certifying exam.

2. See this module's *Problem Supplement*, Problem 12.

3. See this module's *Problem Supplement*, Problem 13.