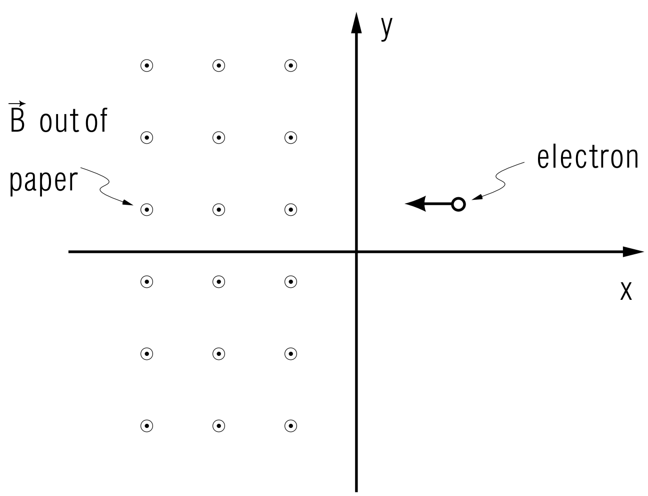
MISN-0-122

FORCE ON A CHARGED PARTICLE IN A MAGNETIC FIELD



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

FORCE ON A CHARGED PARTICLE IN A MAGNETIC FIELD
by
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Input Skills:

1. Calculate the magnitude and direction of the vector product of two given vectors (MISN-0-2).
2. Calculate the force on a charged particle due to a given electric field (MISN-0-115).
3. Calculate the centripetal acceleration and angular velocity of a given particle in uniform circular motion (MISN-0-17).
4. Calculate the energy of a charged particle in a given electric field (MISN-0-117).

Output Skills (Knowledge):

K1. Vocabulary: Lorentz force, magnetic field, magnetic force, magnetic induction, tesla.

K2. State the expression for the Lorentz force; define each quantity.

K3. Derive the expression for the angular velocity and orbital radius of a charged particle moving in a static magnetic field.

K4. Explain why a static magnetic field can neither slow down nor speed up a charged particle.

K5. Explain how crossed magnetic and electric fields may be used as a velocity selector.

Output Skills (Problem Solving):

S1. Given a particle's initial position, velocity and charge, draw a diagram of its motion, including direction, in a given magnetic field.

S2. Given a particle's initial velocity (or energy), mass and charge, calculate its angular velocity and orbital radius in a given magnetic field.

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by

Orilla McHarris

1. Introduction

In a sense, most of us need no introduction to magnets themselves. They are the small things that hold little plastic decorations to people's refrigerators, the large things that pick up huge piles of scrap metal, and the bell part of doorbells - among many other examples. No matter what the size or shape of a magnet, it always has two "ends" - literally the two ends of a bar magnet, or the two "feet" of a horseshoe magnet, or the top side and bottom side of circle of current carrying wire - that can be identified as a "north pole" and a "south pole" (see Fig. 1). In the region around a magnet there is a "magnetic field." This is described by a vector quantity whose direction at any point is given by the tangent to a (often curved) line running from the north pole of the magnet (and then continuing back through the magnet from south pole to north). The exact shapes of these lines for a given magnet can be calculated or measured; some examples are just sketched in here to emphasize the idea of a field surrounding a magnet. Later on we will make calculations assuming a uniform magnetic field. Notice that this implies being in a region such as the one between the poles of the bent horseshoe, or somewhere where the field is constant in magnitude and direction over an area larger than the other dimensions in the particular problem. In this module we are interested in the effect of a magnetic field upon a charged particle. Just as an electric field exerts a force on a charged particle,[1] now we will find

[1]See "Point Charge: Field and Force" (MISN-0-115).

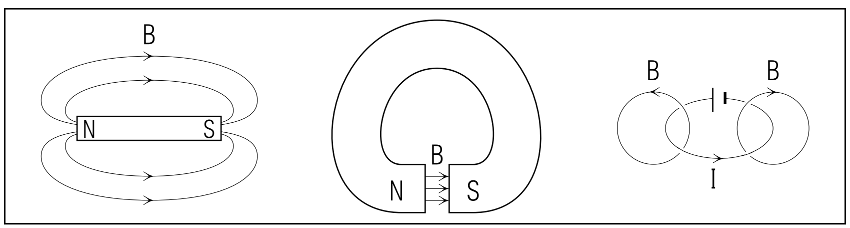


Figure 1. Examples of magnetic fields and their sources.

that a magnetic field can also exert a force on a charged particle, although this "magnetic force" is usually quite different from the electric one and is zero unless the charged particle is moving.

2. The Lorentz Force

2a. The Magnetic Induction Vector. The field in which we are interested is called the "magnetic induction," $\vec{B}$.[2] It is the magnetic counterpart of the electric field $\vec{E}$; but, unlike $\vec{E}$, the magnetic induction exerts a force only on moving charges—and then only on moving charges that have some component of velocity perpendicular to $\vec{B}$. The force on charged particles in a magnetic field is given by:[3]

$$\vec{F}_B = q\vec{v} \times \vec{B}, \tag{1}$$

Where: q = the charge of the particle
$\vec{v}$ = the velocity of the particle; and
$\vec{B}$ = the magnetic induction

The vector product used in Eq. (1), identified by the symbol "×," is used in many parts of Mechanics, such as in the study of torques and angular momentum.[4] Notice that if the particle in question is negatively charged (for example, an electron) a minus sign will come in with the charge and the force will be in the direction opposite to the force on a positive charge with a similar velocity.

2b. Units of Magnetic Induction. This force on a moving charged particle can be used as a measure of the magnetic induction $\vec{B}$, and from it one can see what the units of $\vec{B}$ are:

$$\frac{\text{force}}{\text{charge-velocity}} = \frac{\text{mass-acceleration}}{\text{charge-velocity}} = \frac{\text{mass}}{\text{charge-time}}.$$

[2]Some textbooks also call $\vec{B}$ the magnetic flux density. Colloquially, $\vec{B}$ is often called just "the magnetic field." However the specific name magnetic induction is used to distinguish $\vec{B}$ from other magnetic field vectors.

[3]For the magnetic force on charged particles in an electric current, see "Force on a Current in a Magnetic Field" (MISN-0-123). This force is the basis for the operation of electric motors. For the converse - the production of a magnetic field by a moving charge, see "The Magnetic Field of a Moving Charge" (MISN-0-124) wherein it is seen that a moving charge exerts a magnetic force on other charges and currents.

[4]See "Sums, Differences, and Products of Vectors," (MISN-0-2). The vector product is also reviewed in this module's appendices.

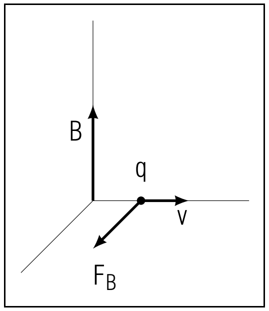


Figure 2. Vector relationships for the magnetic force.

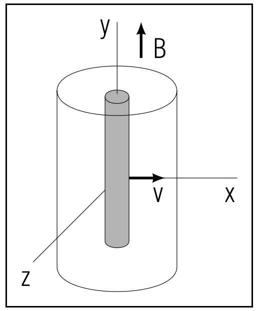


Figure 3. A diagram for a vacuum gauge.

The SI units of $\vec{B}$ are given the special name "tesla," abbreviated T.[5]

$$\mathrm{T} = 1\,\mathrm{kg}/(\mathrm{C\,s})$$

Some books call 1 T a "weber/m^2," although the name tesla is more contemporary. In addition, in scientific literature, B is still sometimes given in terms of the cgs unit "gauss": T $= 10^4$ gauss.

2c. Electric Plus Magnetic Force: The Lorentz Force. If we combine the forces on a charged particle due to both an electric field $\vec{E}$ and a magnetic field $\vec{B}$, we obtain:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}. \qquad (2)$$

This is called the "Lorentz force."[6]

2d. Direction of Lorentz Force. The quantities $\vec{F}$, $\vec{E}$, $\vec{B}$, and $\vec{v}$ are all vector quantities, and it is important to keep track of their directions. The vectors $\vec{F}_E$ and $\vec{E}$ are in the same direction (or exactly opposite direction, if q is negative), but $\vec{F}_B$ and $\vec{B}$ are not. A vector product always involves three mutually perpendicular directions (see Fig. 2). A magnetic field exerts a force on a moving charged particle in proportion to that particle's component of velocity perpendicular to $\vec{B}$, and then the force is in the third direction. Take the example of a particular kind of

[5]Nikola Tesla, Serbian-American Inventor and contemporary of Edison, was awarded 112 U.S. patents during his lifetime, among them being patents for the power transmissions of alternating current. (Edison preferred d.c.)

[6]Hendrick Antoon Lorentz, Nobel-Prize-winning Dutch theoretical physicist, made many important contributions to electromagnetic and relativity theory.

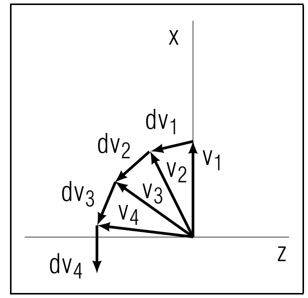


Figure 4. Vector diagram of the change in velocity of a charged particle in a 38magnetic field.

vacuum gauge that makes use of a magnetic field (see Fig. 3). The gauge is enclosed in a cylindrical glass bulb that looks much like an electronic vacuum tube. Down the center of the cylinder runs a hot wire. Let us take that wire as the y axis of our coordinate system. Assume the cylinder sits in a uniform magnetic field directed parallel to the y-axis of our coordinate system. What will happen to an electron boiled off the wire at $y = 0$ in the x-direction? From the definition of the vector product, and keeping in mind the fact that the electron has a negative charge, we find that the electron that started off in the x-direction is now "pushed" in the negative z-direction.

3. Motion In a Uniform Magnetic Field

3a. Speed Unchanged in the Magnetic Field. The magnitude of a particle's velocity is not changed by a magnetic field since the magnetic field does no work on the particle. This is obvious from the definition of work:

$$W = \int_A^B \vec{F} \cdot d\vec{s}, \qquad (3)$$

and $d\vec{s}$ is parallel to the velocity $\vec{v}$. Since the force due to the magnetic field is always perpendicular to $\vec{v}$, it can never do work on a charged particle and thus can never change the energy or the speed of the particle.

3b. Direction of Particle's Motion is Changed. Although $\vec{F}_B$ cannot slow down or speed up a particle, it does constantly change a particle's direction of motion. Take an electron traveling with an initial velocity $\vec{v} = v\hat{x}$ in a magnetic field $\vec{B} = B\hat{y}$. Then $\vec{F}_B$, which acts always at right angles to the electron's direction of motion, pushes the electron constantly to its left in the x-z plane. The electron can never acquire a component of velocity in the y-direction since $\vec{B}$ is in the y-direction. The easiest way to visualize what the path of the electron looks like is to take repeated vector sums: if

$$\begin{aligned}\vec{v}_1 &= \text{the electron's initial velocity,}\\ d\vec{v}_1 &= \text{the change in the electron's velocity as it}\\ &\quad\ \text{first feels the effect of } \vec{F},\\ \vec{v}_2 &= \text{the ``new'' velocity, } \vec{v}_1 + d\vec{v}_1,\\ d\vec{v}_2,\, d\vec{v}_3,\, \ldots &= \text{successive small changes in velocity,}\\ \vec{v}_2,\, \vec{v}_3,\, \ldots &= \text{successive ``new'' velocities,}\end{aligned}$$

then the vector diagram for the electron's change in velocity looks like the one shown in Fig. 4: that is, the electron moves in a circle in the x-z plane. Thus the motion of a charged particle in a uniform magnetic field is an example of uniform circular motion, and we can calculate such quantities of interest as the radius of the circle and the frequency of revolution.

3c. Uniform Circular Motion in a Magnetic Field. The magnetic field provides the centripetal force necessary for uniform circular motion. The expression for centripetal force is given by:[7]

$$F = \frac{mv^2}{R} \tag{4}$$

where v is the orbital speed and R is the orbital radius.

In a magnetic field the magnitude of the centripetal force is

$$F_B = q\, v_\perp B, \tag{5}$$

where $v_\perp$ is the component of $\vec{v}$ perpendicular to $\vec{B}$.

Notice that $v_\perp$ is the orbital speed of the charged particle in a magnetic field. Thus we have

$$\frac{mv_\perp^2}{R} = qv_\perp B \tag{6}$$

or

$$R = \frac{mv_\perp}{qB}. \tag{7}$$

For uniform circular motion we also know the angular speed is given by:

$$\omega = \frac{v_\perp}{R}$$

[7]See "Uniform Circular Motion: Centripetal Force, Banking Angles, Centrifugal Force" (MISN-0-17).

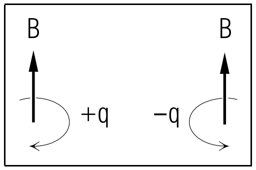


Figure 5. The trajectories of charged particles in magnetic fields: (a) clockwise around the field vector for a positive charge; and (b) counterclockwise around the field vector for a negative charge.

so that in a magnetic field

$$\omega = \frac{qB}{m}. \tag{8}$$

It is often of interest to know also the direction in which a given particle will turn in a given magnetic field. One can determine this for each case by using the equation $\vec{F} = q\vec{v} \times \vec{B}$, or one can notice that a positively charged particle will always move clockwise around the axis defined by $\vec{B}$ whereas a negatively charged particle will always move in a counterclockwise direction around $\vec{B}$, assuming the magnetic field is pointing at the observer (see Fig. 5). One other thing to notice is that ω depends on B but not on either R or v. It was this observation that led to the construction of the first cyclotron.[8]

3d. Helical Motion in a Magnetic Field. If the particle in question has a component of velocity parallel as well as perpendicular to $\vec{B}$, the trajectory of the particle will be helical. So far we have considered the example of a charged particle with all of its velocity in a plane perpendicular to $\vec{B}$, although where it was necessary we have been careful to write equations in terms of $v_\perp$ so that they will still hold true even for a charged particle with a component of velocity in the $\vec{B}$ direction. Suppose an electron with a velocity

$$\vec{v} = v_x\hat{x} + v_y\hat{y} \tag{9}$$

entered a region of space where the magnetic field is $\vec{B} = (3\,\mathrm{T})\hat{y}$. Then v_x would be the $v_\perp$ we have been talking about and the magnetic force

$$\vec{F}_B = qv_x\hat{x} \times \vec{B} \tag{10}$$

would still cause the electron to start turning in a circle. The y-component of $\vec{v}$ would be unaffected by $\vec{B}$, however, and would remain a constant of the electron's motion. Thus the electron would exhibit helical motion around a direction parallel to the y-axis. In general, a charged particle with a component of velocity in the direction of a magnetic field will exhibit helical motion around an axis by $\vec{B}$.

[8]See "Trajectory of a Charged Particle in a Magnetic Field: Cyclotron Orbits (Computer Project)" (MISN-0-127).

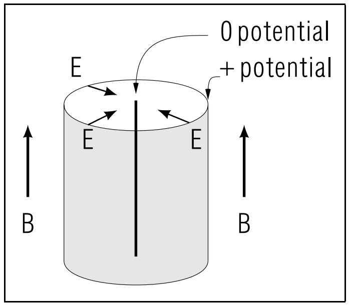


Figure 6. A vacuum gauge employing both magnetic and electric fields.

4. Combined Electric, Magnetic Fields

4a. Vacuum Gauge. Our example of a vacuum gauge may be used to consider what happens when both electric and magnetic fields are present at the same time. That is, we can calculate the action of the Lorentz force on a moving charged particle. This particular gauge is essentially a diode: the wire down the center (the cathode) is held at ground (zero) potential, while a cylinder around the wire (the anode) is held at a positive potential. Thus an electron leaving the wire is accelerated outward toward the cylinder by the electric force, $\vec{F}_E = e\vec{E}$. In order to make the gauge more efficient, however (so that the electron will travel on a longer path and have a chance to ionize more atoms in the "vacuum"), it is placed in a magnetic field with $\vec{B}$ parallel to the central wire (see Fig. 6). We have already seen that the magnetic field alone would make the electron move in circles (or a helix) around the wire. Combining the outward motion due to the electric field and the circular motion due to the magnetic field, we find that the electron spirals out from the wire in widening circles until it strikes the cylinder. If the radius of the cylindrical anode is 12 cm and the potential of the anode is 300 V, let us calculate how strong $\vec{B}$ should be so that at its final velocity the electron just touches the anode. We know that:

$$B = \frac{mv_\perp}{qR} \tag{11}$$

We can calculate the final speed from the anode potential:[9]

$$\frac{1}{2}mv_\perp^2 = qV \tag{12}$$

or

$$v_\perp = \left(\frac{2qV}{m}\right)^{1/2} \tag{13}$$

[9]See "Particle Energy in an Electrostatic Field" (MISN-0-117).

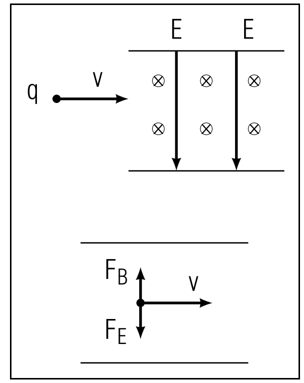


Figure 7. Crossed electric and magnetic fields used as a velocity selector

Thus

$$B = \frac{1}{R}\left(\frac{2mV}{q}\right)^{1/2}. \tag{14}$$

The mass and charge of an electron are $m = 9.11 \times 10^{-31}$ kg and $q = 1.6 \times 10^{-19}$ C, respectively, so

$$B = \frac{1}{0.12m}\left(\frac{(2)(9.11 \times 10^{-31}\,\mathrm{kg})(300V)}{1.6 \times 10^{-19}\,\mathrm{C}}\right)^{1/2} = 4.87 \times 10^{-4}\,\mathrm{T}.$$

Now it is easy to calculate the frequency with which the electron circles the central wire (remember it is independent of energy):

$$\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi}\frac{qB}{m} \tag{15}$$

$$\nu = \frac{1}{2\pi}\frac{(1.6 \times 10^{-19}\,\mathrm{C})(4.87 \times 10^{-4}\,\mathrm{T})}{9.11 \times 10^{-31}\,\mathrm{kg}}$$

4b. Velocity Selector. By appropriately choosing the electric and magnetic fields in a system, the magnetic and electric forces will cancel one another for particles moving at a specific velocity. Notice that the Lorentz force on a moving charged particle will be zero if $q\vec{E} = -q\vec{v} \times \vec{B}$, and the particle will move in a straight line (assuming no other forces are acting on it). The phrase "crossed fields" is used to describe this situation because $\vec{E}$ and $\vec{B}$ must be at right angles to each other as well as to $\vec{v}$ in order for $q\vec{E} = -q\vec{v} \times \vec{B}$. Figure 7 illustrates this situation. A positively charged particle is sent into a region where, for example, $\vec{E}$

is directed downward and $\vec{B}$ is directed into the paper.[10] The resulting forces are in opposite directions and will exactly cancel when $v = E/B$. Adjustment of the $\vec{E}$ and $\vec{B}$ fields for no deflection of a given particle can be used in two ways: If v is unknown, E and B can be changed until the particle moves in a straight line, and then v can be calculated from the measured magnitudes of $\vec{E}$ and $\vec{B}$. If on the other hand, one wants to select particles with a given velocity out of a group of particles with a range of velocities, $\vec{E}$ and $\vec{B}$ can be set such that only those particles with the desired velocity will pass straight through the velocity selector, while all the other particles will bend off to the side or into the walls of the equipment.

Acknowledgments

Julius Kovacs made a number of useful suggestions. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

Glossary

- **Lorentz force**: the net force $\vec{F}$ on a particle with charge q moving with velocity $\vec{v}$ due to the electric and magnetic fields, $\vec{E}$ and $\vec{B}$, at the particle's space-point: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$.
- **magnetic field**: a vector quantity responsible for a force on a moving charged particle. For a magnet, the direction of the field at any point is given by the tangent to a line running from the "north pole" to the "south pole" of the magnet. The term "magnetic field" is somewhat ambiguous since it is also sometimes used for both $\vec{B}$, technically the "magnetic induction," and for $\vec{H}$, technically the "magnetizing force." The SI unit of the magnetic (induction) field is the tesla.
- **magnetic force**: the force on a moving charged particle due to a magnetic field, specified by the equation $\vec{F}_m = q\vec{v} \times \vec{B}$, where the magnetic field $\vec{B}$ is called the "magnetic induction."

[10]Notice, if you have not seen it before, the convention for drawing a vector into or out of the paper: for a vector pointed into the paper one draws an "$\times$" as if we were seeing the feathers on the back of an arrow flying away from us, while for a vector pointed out of the paper, one draws a "$\cdot$" as if the tip of the arrow were coming at us.

- **magnetic induction**: the technical name for the magnetic field, $\vec{B}$, defined through the characteristic magnetic force on a moving charge.
- **tesla**: the SI unit of magnetic induction: abbreviated T; T = kg/(C s).

A. The Geometric Definition of the Vector Product

The vector product of two arbitrary vectors $\vec{A}$ and $\vec{B}$ is defined as the vector quantity whose magnitude is given by the product of the magnitudes of the two vectors times the sine of the angle between the vectors when they are placed "tail-to-tail," and whose direction is perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$. The vector product (also referred to as the "cross product") is denoted by $\vec{A} \times \vec{B}$. The magnitude of the product may be written as:

$$|\vec{A} \times \vec{B}| = AB \sin\theta.$$

There are, however, two directions that are perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$. The correct direction may be chosen by applying the "right-hand rule": "Rotate vector $\vec{A}$ into vector $\vec{B}$ through the acute (smaller) angle between their directions when they are placed tail-to-tail. Follow this rotation with the curled fingers of your right hand, and the direction of your extended thumb identifies the direction of the vector product." This rule is sufficient to distinguish between the two possible choices for the direction of a vector product. Notice that the order of multiplication in vector products is very important. The product $\vec{B} \times \vec{A}$ has the same magnitude as $\vec{A} \times \vec{B}$, but the directions of the two products are opposite. In general:

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}.$$

We say that vector products do not "commute," or that the vector product is a "noncommutative" operation.

If two vectors are parallel, the angle between their directions is zero, so by the definition of the magnitude of vector products their cross product is zero. Similarly, if two vectors are perpendicular, the angle between their directions is 90°. Since sin90° = 1, the magnitude of the vector product of the two is just the product of their magnitudes, and the direction of the vector product is determined by the right-hand rule. By applying these observations to the vector product of the cartesian unit vectors $\hat{x}$, $\hat{y}$ and $\hat{z}$, we may derive the following useful relations:

$$\hat{x} \times \hat{x} = \hat{y} \times \hat{y} = \hat{z} \times \hat{z} = 0$$

$$\hat{x} \times \hat{y} = -\hat{y} \times \hat{x} = \hat{z}$$
$$\hat{y} \times \hat{z} = -\hat{z} \times \hat{y} = \hat{x}$$
$$\hat{z} \times \hat{x} = -\hat{x} \times \hat{z} = \hat{y}.$$

These relations are used in the algebraic definition of vector products.

B. The Algebraic Definition of the Vector Product

If we express vectors $\vec{A}$ and $\vec{B}$ in their cartesian component form, the vector product of $\vec{A}$ and $\vec{B}$ may be written:

$$\vec{A} \times \vec{B} = (A_x\hat{x} + A_y\hat{y} + A_z\hat{z}) \times (B_x\hat{x} + B_y\hat{y} + B_z\hat{z}).$$

If this expression is expanded algebraically as we would the product $(x+3)\cdot(2x-5)$, except that the cross product is used instead of scalar multiplication, then this vector product may be expressed as a combination of the cartesian components of $\vec{A}$ and $\vec{B}$ and cross products of the cartesian unit vectors. Using the relations between the cartesian unit vectors developed in Appendix A and denoting the vector product as a third vector $\vec{C}$, we may write:

$$\vec{C} = \vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\hat{x} + (A_zB_x - A_xB_z)\hat{y} + (A_xB_y - A_yB_x)\hat{z}$$

or:

$$C_x = A_yB_z - A_zB_y,$$
$$C_y = A_zB_x - A_xB_z,$$
$$C_z = A_xB_y - A_yB_x.$$

The mnemonic for remembering the order of the subscripts on these components is to note that, starting from left to right, the first three subscripts in each of the three equations for the components of $\vec{C}$ are always cyclic permutations of xyz (xyz, yzx, zxy). Another way to remember the order of combination of the unit vectors and the components of $\vec{A}$ and $\vec{B}$ is to use this determinant:

$$\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Expansion of this determinant leads to the same expression for $\vec{C}$ derived earlier.

▷ Show that the vector product of $\vec{A} = 5\hat{x} - 2\hat{y}$ and $\vec{B} = \hat{x} + \hat{y} + 3\hat{z}$ has these components:

$$C_x = -6,\ C_y = -15,\ C_z = 7.$$

C. The Direction of the Vector Product

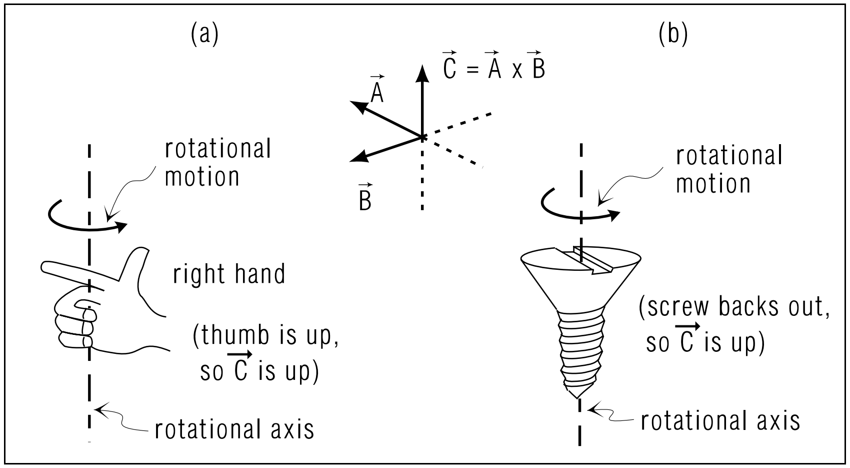
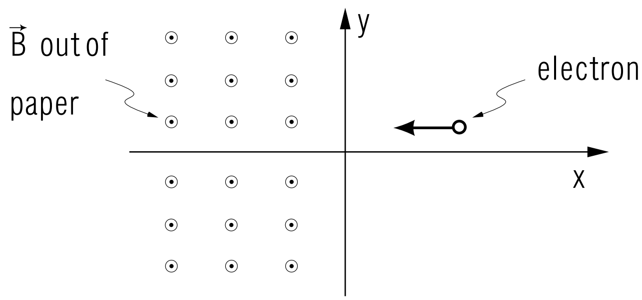


Figure 8. Two rules for finding the direction of $\vec{C} = \vec{A} \times \vec{B}$ by: (a) the "right hand" rule; (b) the "screw" rule.

PROBLEM SUPPLEMENT

Note: Problem 7 also occurs in this module's *Model Exam*.



An electron with charge -1.6×10^{-19} C and mass 9.1×10^{-31} kg has an initial position in the x-y plane shown in the diagram and an initial velocity $-4.8\,\mathrm{m/s}\,\hat{x}$. The magnetic field is zero for $x > 0$, $0.032\,\mathrm{T}\,\hat{z}$ for $x < 0$. Note: Until you are told otherwise in a magnetic-force problem, assume that $\vec{E} = 0$. Of course the electron *has* an electric field but it does not act on the electron itself via the Lorentz force.

1. Sketch the electron's trajectory on the diagram. *Help: [S-1]* Mark the parts of the trajectory that are *straight lines* as such, and mark the parts of the trajectory that are *circular arcs* as such.

2. Starting from the Lorentz force, calculate the radius of curvature for any curved portion of the trajectory.

3. Starting from the Lorentz force, calculate the direction and magnitude of the electric field which would exactly cancel the magnetic force on the electron. Add the resulting trajectory to the diagram and label it. *Help: [S-3]*

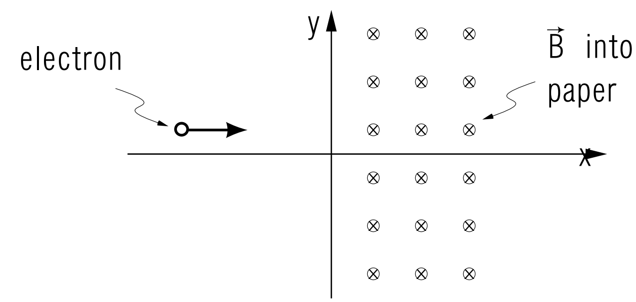
4. Suppose a number of electrons were at the same initial position and moving in the same initial direction shown in the sketch, but with a range of velocities. Explain how the results of Problem 3 could be used to select those electrons which have the same initial velocity as the electron in Problems 1-3.

5. Suppose the particle was not an electron but a proton, a particle of opposite charge to that of the electron and with a mass approximately

2000 times larger. Compare all aspects of this proton's trajectory to those of the electron in Problems 1-3.

6. Suppose the electron in the Problems 1-3 also had a z-component in its initial velocity. Describe the effect on its trajectory.

7.



An electron with charge -1.6×10^{-19} C and mass 9.1×10^{-31} kg has an initial position in the x-y plane shown in the diagram and an initial velocity $\vec{v} = (2 \times 10^5\,\mathrm{m/s})\hat{x} - (6 \times 10^5\,\mathrm{m/s})\hat{z}$. The magnetic field is zero for negative x but is 0.01 T, into the paper, for positive x.

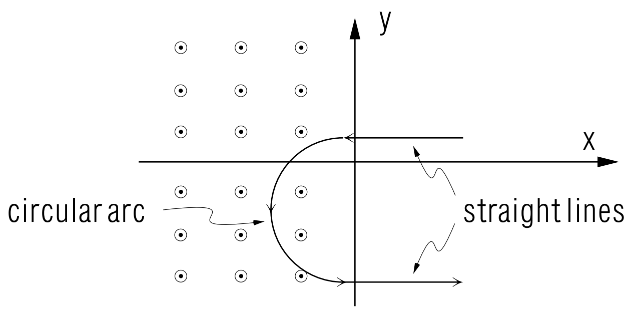
a. Sketch the trajectory of the electron, as projected onto the x-y plane, on the above diagram. *Help: [S-2]* Describe the 3-dimensional motion.

b. Starting from the Lorentz force, calculate the radius of curvature for any curved portion of the trajectory's projection onto the x-y plane. Use the projection of the trajectory onto the x-y plane.

c. Starting from the Lorentz force, calculate the direction and magnitude of the electric field which would exactly cancel the magnetic force on the electron. *Help: [S-4]* Add the resulting trajectory to the diagram and label it appropriately.

Brief Answers:

1.



2. 0.85×10^{-9} m.

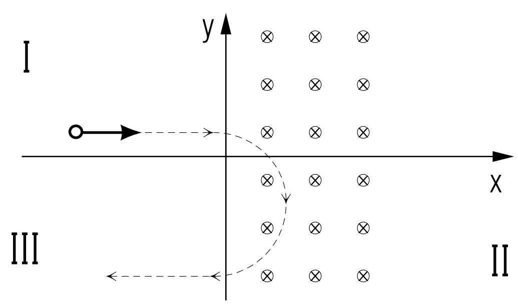
3. $\vec{E} = -0.15\,\mathrm{N/C}\,\hat{y}$ for $x \leq 0$, zero elsewhere: then electron is undeflected and stays on original path. *Help: [S-3]*

4. Electrons with higher initial speed are deflected downward, lower initial speed upward.

5. Radius of curved trajectory is 2000 times larger and trajectory curves in opposite direction.

6. The electron's trajectory in the region of the $\vec{B}$-field is a half turn of a helix. Its trajectory is the line traced out by a point which moves around a cylindrical axis at constant radius and rate while it advances along the axis at a constant rate. Thus the trajectory consists of a curve on a cylindrical surface.

7. a.



Region I: straight line in a plane parallel to the x-z plane.

Region II: helical path with the axis of the helix along the z-axis.

Region III: straight line in a plane parallel to the x-z plane.

b. $R = 0.114\,\mathrm{mm}$

c. $\vec{E} = -2000\hat{y}\,\mathrm{N/C}$; Trajectory is a straight line from left to right.

SPECIAL ASSISTANCE SUPPLEMENT

S-1 *(from PS, problem 1)*

1. Look at the sign of the electron's charge and look at Fig. 5.
2. Note that no scales are drawn on the figure (on the x- and y-axes) so you can draw the trajectory any size you want. If meters, say, had been marked off along the axes, you would have had to use them and plot your trajectory to scale.
3. A "trajectory " is the particle's "path," its "trail." You sketch it by: (i) putting your pen or pencil point on the paper at the position of the particle; and (ii) moving the point along the paper, following the path the particle will take.

S-2 *(from PS, problem 7a)*

This module's *text* describes how to deal with $\vec{v}$ having a component not prependicular to the field. Note that $\hat{x} \times \hat{y} = \hat{z}$, so $\hat{z}$ is out of the paper (this is called a "right hand coordinate system.")

S-3 *(from PS, problem 3)*

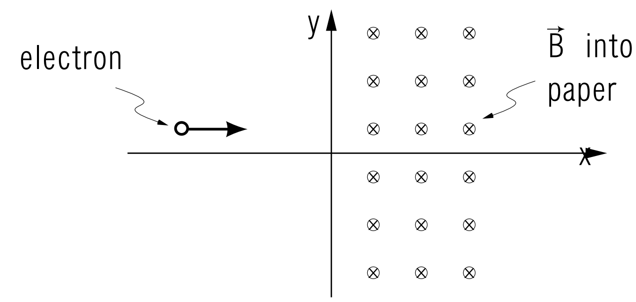
Why must $\vec{E}$ be *downward*? Work the magnetic field part of the Lorentz force and see that it exerts a *downward* force on the negatively charged electron. Then to cancel that downward magnetic force on the electron we must apply an *upward* electric force. An upward electric force, on a negatively charged particle like the electron, is exerted by a *downward* electric field. With the two fields present, the total force on the electron is zero and hence its acceleration is zero. That implies a constant velocity which means a straight-line trajectory.

S-4 *(from PS, problem 7c)*

If you have trouble with this problem, go back and do problem (3) properly. Then you should be able to do this problem easily.

MODEL EXAM

$$\mathrm{T} = \mathrm{kg\,C^{-1}\,s^{-1}}$$

1. See Output Skills K1-K5 in this module's *ID Sheet*.
2. 

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c. Starting from the Lorentz force, calculate the direction and magnitude of the electric field which would exactly cancel the magnetic force on the electron. Add the resulting trajectory to the diagram and label it appropriately.

Brief Answers:

1. See this module's *text*.
2. See Problem 7 in this module's *Problem Supplement*.